Contagion and Efficiency in Gross and Net Interbank Payment Systems*

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The increased fragility of the banking industry has generated growing concern about the risks associated with payment systems. Although in most industrial countries different interbank payment systems coexist, little is really known about their properties in terms of risk and efficiency. How should payment systems be designed? We tackle this question by comparing the two main types of payment systems, gross and net, in a framework where uncertainty arises from several sources: the time of consumption, the location of consumption, and the return on investment. Payments across locations can be made either by directly transferring liquidity or by transferring claims against the bank in the other location. The two mechanisms are interpreted as the gross and net settlement systems in interbank payments. We characterize the equilibria in the two systems and identify the trade-off in terms of safety and efficiency. Journal of Economic Literature Classification Numbers: G21, E51.

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1. INTRODUCTION

The impressive growth in the value of daily interbank payments\(^1\) has raised concerns about the potential systemic risk induced by contagion. This effect, also known as the domino effect, occurs if the failure of a large financial institution to settle payment obligations triggers a chain reaction that threatens the stability of the financial system. (See, among others, Brimmer (1989).) The two main types of large-value interbank payment systems, “net” and “gross,” differ sharply in their exposure to contagion risk. In the former, netting the positions of the different banks through compensation of their claims only at the end of the day implies intraday credit from one bank to another, and exposes banks to contagion.\(^2\) In the latter, transactions are typically settled irrevocably on a one-to-one basis in central bank money, so that banks have to hold large reserve balances in order to execute their payment orders.

Technological improvements in data transmission have substantially lowered the transaction cost of settling in gross payment systems, but netting still significantly economizes on the liquidity that banks have to transfer. Thus an important question is: how do gross and net interbank payment systems compare from a cost–benefit standpoint? In particular, how do the characteristics of these systems compare under different environments? Our objective in this paper is to address these questions.

A number of factors must be taken into account when the trade-offs between alternative settlement mechanisms are evaluated: collateral needs, use of information about bank solvability, and above all liquidity needs and contagion risk.\(^3\) Our aim is to address what happens when we derive the liquidity costs and the contagion costs endogenously from the liquidation of long-term projects.

To tackle these issues we construct a general equilibrium model based on Diamond and Dybvig (1983) (D–D). Unlike D–D, where consumers’ uncertainty arises only from the time of consumption, in our model consumers are also uncertain about the location of their consumption. Consumers’ geographic mobility generates payment needs across space. Financial inter-

\(^1\) The average daily transactions on the U.S. payment systems C.H.I.P.S. and FEDWIRE have grown from $148 and $192 billion, respectively, in 1980 to $885 and $796 billion in 1990. (See Rochet and Tirole (1996b).)

\(^2\) Average payments made by large banks can be tenfold the capital of smaller banks in the netting process. Because of this difference in size the failure of a single large participant, even if its own net credit position does not threaten the settlement system, could lead small banks to have settlement obligations greater than the amount of their capital. See the simulations by Humphrey (1986). Similar simulations by Angelini et al. (1996) for the Italian system yield a much smaller impact of systemic risk.

\(^3\) See, e.g., Rochet and Tirole (1996b).
mediaries are justified on two accounts: their insurance role, as in D–D, and their role in transferring property rights, as in Fama (1980). Payments across locations can be made by transferring either liquid assets (gross system) or claims against the intermediary in the location of destination (net system). The choice of a net or gross payment system affects equilibrium consumption, and we can compare the two systems by measuring ex ante expected utility.

A key assumption in our analysis is that investment returns are uncertain and some depositors have better information on returns. If investment returns were certain, there would be only speculative bank runs, as in D–D. In this case, which we examine only as a benchmark, we show that netting dominates. But in a setting with asymmetric information and uncertain returns, information-based (or fundamental) runs can occur, as in Chari and Jagannathan (1988) and Jacklin and Bhattacharya (1988). In this richer and more realistic setting, the issue of the trade-off between gross and net systems arises.4

A first contribution of this paper is to identify the equilibria under gross and net systems. Since under the gross system banks are not linked to each other, the equilibria simply correspond to those of isolated islands. Under netting, on the other hand, since banks are linked through intraday credits, the failure of one bank may affect the payoff of depositors in another. This feature of netting generates two equilibria, both inefficient. In the first there are no bank runs; banks net their claims, and are therefore exposed to contagion. We call this the potential contagion equilibrium. Because with netting each bank has claims on the assets of the other banks, when one goes bankrupt others are affected. In the second equilibrium, consumers rationally anticipate the potential effect of contagion on future consumption and optimally decide to run on their bank. We call this the contagion-triggered bank run equilibrium.

The main contribution of the paper is the analysis of the trade-off between gross and net systems. Our results are consistent with the intuition that a gross system is not exposed to contagion but makes intensive use of liquidity, while a net system economizes on liquidity but exposes banks to contagion. We show when, depending on the values of model parameters, a particular system is preferred. A gross payment system is preferred if the probability of banks having a low return is high, if the opportunity cost of holding reserves is low, and if the proportion of consumers that have to consume in another location is low. Otherwise a net system dominates.

There is an incipient literature on payment systems which helped us derive our modeling framework. McAndrews and Roberds (1995) model

4 Using data from the national banking era (1863–1914), Gorton (1988) shows that the majority of bank panics were in fact information-based.
bank payment risk using the D–D framework of speculative runs. They focus on the banks’ demand for reserve provisions and consider a multilateral net settlement system where claims on a bank are valid only if reserves are transferred to the recipient bank. Kahn and Roberds (1996a) analyze a gross settlement system where adverse selection gives rise to the Akerlof effect because banks with above average assets prefer cash settlements to debt settlements. Using an inventory-theoretic framework to model the trade-off between safety and efficiency, Kahn and Roberds (1996b) find that netting economizes on reserves but increases moral hazard because it gives the banks additional incentives to default. Rochet and Tirole (1996a) model interbank lending to address the issue of contagion and the “too-big-to-fail” policy.

The paper is organized as follows. Section 2 reviews the main institutional aspects of the payment systems. Section 3 sets up a benchmark model of the payment system without investment return uncertainty. In Sections 4 and 5, which are the core of our analysis, we introduce private information on future uncertain returns and compare the equilibria under the different mechanisms. Section 6 discusses some policy implications. Section 7 suggests extensions. Section 8 concludes.

2. INSTITUTIONAL ASPECTS OF THE INTERBANK PAYMENT SYSTEM

Time Dimension of Settlement

The organization of interbank payment systems revolves around the time dimension of the final settlement of transactions. It is convenient to begin our discussion by introducing two stylized alternative mechanisms, gross and net, both with settlement in central bank money. The risk and liquidity characteristics of the two mechanisms are the object of our analysis.

The gross mechanism achieves immediate finality of payment at the cost of intensive use of central bank money. Bilateral and multilateral netting economize on the use of central bank money, essentially by substituting explicit or implicit interbank intraday credit for central bank money. A difference between a net settlement system and an interbank money market is that risk is priced in an interbank market and rationing of a particular bank may occur, triggered by bad news on its solvency. This does not happen under netting, where implicit credit lines are automatically granted.

The very fact that netting economizes on central bank money allows a participant both to accumulate large debt positions during the day, perhaps exceeding its balances in central bank money, and to accumulate claims on other banks that may exceed its own capital. In fact, if no incoming
payment decreases a bank’s exposure, the interbank market will usually provide the necessary central bank money. A settlement risk arises when a participant has insufficient funds to settle its obligations when they are due. In this case different procedures are followed in the various systems, ranging from deleting the orders with insufficient funds and recalculating the balances (unwinding), to queuing them.\(^5\) As a result of settlement risk, netting also increases systemic risk, because of the possible contagion effects. Contagion is possible when unwinding of orders takes place, because participants that had been net creditors of the failed institution may have sent order payments on the basis of the expected funds that are not forthcoming.\(^6\)

Settlement risk also arises when commercial bank money is used to settle, as the finality of the transaction is then delayed by definition.

Before the widespread diffusion of telematic technology, the systems most often encountered were gross mechanisms settled in commercial bank money and net mechanisms settled in central bank money. Due to its high liquidity cost, gross settlement in central bank money was practically never used (Padoa-Schioppa (1992)). The advent of telematics has made possible another mechanism, namely real-time gross settlement in central bank money. The innovation stems from the real-time feature, which dramatically increases the velocity of circulation of deposits at the central bank and thus reduces the opportunity cost of using central bank money. The net systems have also been influenced by telematics as recent technological developments that increase the amount that can be netted per unit of reserve increase both settlement and systemic risk. To this it must be added that multilateral netting is subject to moral hazard since banks perceive that the central bank will prevent systemic collapses. In balance, telematics has altered the trade-off between cost and risk, making real-time gross settlement systems relatively cheaper. In fact, its use is now encouraged by several central banks.

Models of Interbank Payment Systems

The large-value interbank payment systems currently operational can be grouped into three general models (Horii and Summers (1993)): (i) Gross settlement operated by central bank with explicit intraday credit (e.g., FEDWIRE); (ii) Gross settlement operated by central bank without intraday credit (e.g., Swiss Interbank Clearing System (S.I.C.)); (iii) Deferred net

\(^5\) In the Swiss system, when there are insufficient funds in the originator’s account, orders are queued until sufficient funds have accumulated in that account and may be canceled at any time by the originator. Orders still in queue at a prespecified time are cancelled automatically.

\(^6\) For a detailed analysis of systemic risk see Van den Berg and Veale (1993) and for estimates of the consequences of unwinding see Humphrey (1986) and Angelini and Giannini (1994).
multilateral settlement (e.g., Bank of Japan Network, B.O.J.; and Clearing House Interbank Payment System, C.H.I.P.S). See Table 1 for a synthesis of the main features of payment systems.

In the FEDWIRE model, the central bank settles orders payment-by-payment and irrevocably. Insufficient funds in the ordering bank’s accounts result in an extension of explicit intraday credit from the central bank. Credit is provided with the expectation that funds will be deposited in the account before the end of the business day. Meanwhile the central bank bears the settlement risk.

The S.I.C. model is a no-overdraft system in which payment orders are processed on a first-in first-out principle as long as they are fully funded from accounts at the Central Bank. It thus implies real-time computing facilities to execute payments, to prevent the use of intraday credit, and to handle orders with insufficient funds.

In the C.H.I.P.S. model, at designated times during the day payments are multilaterally netted, resulting in one net obligation for each debtor due at settlement time. Implicit intraday credit is extended by the participants, not by the operator of the system, which may be either a private

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### TABLE I
Main Payment Systems, 1992

<table>
<thead>
<tr>
<th></th>
<th>USA</th>
<th>JAPAN</th>
<th>SWITZERLAND</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Starting year</strong></td>
<td>1918</td>
<td>1971</td>
<td>1988</td>
</tr>
<tr>
<td><strong>Gross vs net settlement</strong></td>
<td>gross</td>
<td>net</td>
<td>net&lt;sup&gt;a&lt;/sup&gt;</td>
</tr>
<tr>
<td><strong>Privately vs publicly managed</strong></td>
<td>public: FED</td>
<td>private: NYCHA</td>
<td>public: Bank of Japan</td>
</tr>
<tr>
<td><strong>Intraday central bank credit</strong></td>
<td>Yes</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td><strong>Payment volume per year in million</strong></td>
<td>68</td>
<td>40</td>
<td>64</td>
</tr>
<tr>
<td><strong>Payment value per year in trillion $</strong></td>
<td>199</td>
<td>240</td>
<td>50</td>
</tr>
<tr>
<td><strong>Average payment in million $</strong></td>
<td>3</td>
<td>6.1</td>
<td>33.4&lt;sup&gt;b&lt;/sup&gt;</td>
</tr>
<tr>
<td><strong>Daily payment value in billion $</strong></td>
<td>797</td>
<td>942</td>
<td>1,198</td>
</tr>
<tr>
<td><strong>Number of participants</strong></td>
<td>11,453 banks</td>
<td>20 settling banks</td>
<td>461 banks, securities firms, and brokers</td>
</tr>
<tr>
<td><strong>Procedures in case of failure to settle</strong></td>
<td>if overdraft exceeds cap transaction is rejected or queued</td>
<td>if overdraft exceeds cap, loss shared among participants; settlement is guaranteed</td>
<td>order queued</td>
</tr>
</tbody>
</table>

<sup>a</sup> BOJ-NET offers also a gross settlement system.

<sup>b</sup> Only for the Clearing component.

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clearinghouse or the central bank. Limits are set for intraday credits, both in this and in the FEDWIRE model. Loss-sharing arrangements govern the distribution of losses from settling failures among the members. Regardless of who operates the system, deferred obligations are finally settled in accounts at the central bank.

In what follows, we will mainly focus on gross vs. net payment systems. 7

3. SETUP OF THE MODEL

Basic Model

We consider two identical island-economies, $J = A, B$, with D–D features. Consumers, whose total measure is 1 at each island, are located at A or at B. There is one good and there are three periods: 0, 1, 2. The good can be either stored at no cost from one period to the next or invested. In each island there is a risk-neutral perfectly competitive bank (which can be interpreted as a mutual bank) with access to the investment technology. Each consumer is endowed with 1 unit of the good at time 0. Consumers cannot invest directly but can deposit their endowment in the bank in their island which stores it or invests it for their future consumption. The investment of 1 unit at time 0 returns $R$ at time 2, with $R > 1$, if not liquidated at time 1. If a fraction $\alpha$ of the investment is liquidated at time 1, the return is $\alpha$ at time 1 and $(1 - \alpha)R$ at time 2.

The bank offers depositors a contract that allows them to choose when to withdraw. 8 To finance withdrawals at time 1 the bank liquidates $L$ units so that it receives $R(1 - L)$ at time 2.

As in D–D, consumers are of two types, early diers and late diers. A fraction $t$ die in the first period and $(1 - t)$ die in the second period. However, we modify the D–D model by introducing the additional complexity that late diers face uncertainty at time 1 as to the island in which they will be able to consume at time 2. A fraction $(1 - \lambda)$ of the late diers (the compulsive travelers) can consume only in the other island. The remaining fraction $\lambda$ (the strategic travelers) can consume in either island interchangeably. Nature determines at time 1 which consumers are early diers and which of the late diers are compulsive travelers or strategic travelers. This information is revealed privately to consumers.

7 Since we take a general equilibrium standpoint, the case where central bankers bear the risk (as in FEDWIRE) is left out because it means that the central bank has to levy taxes to fund its rescue operation. Hence, up to a redistribution, we are facing the same issues as in a netting payment system.

8 As is usual in this literature, deposits cannot be traded at time 1.
To analyze how individuals can consume at time 2 in the other island we introduce two payment mechanisms: gross and net.\footnote{To be consistent with Section 2, in terms of the previous classification of interbank payment systems, models (i) as FEDWIRE and (iii) as B.O.J. are “net” because there is no need to liquidate the investment, while (ii) as S.I.C. is “gross.”}

In a gross mechanism, to satisfy the travelers’ demand for the good at time 1 the banks liquidate a fraction of the investment. Then the travelers transfer the good costlessly from A to B or vice versa. The implicit cost of transferring the good across space is the foregone investment return. Since liquidation occurs before the arrival of incoming travelers, their deposits cannot be used to replace those of the departing travelers. In our model a gross system does not allow trade among banks.

The attempt to replace the deposits of the departing travelers with those of the incoming travelers is the rationale behind a netting system. In a netting system, banks are linked by a contract. Under the terms of this contract, member banks extend credit lines to each other to finance the future consumption of the travelers without having to make the corresponding liquidation of investment. The claims on the banks’ assets arising from these credit lines are accepted by all banks in the system. Thus in a net mechanism, the late diers, besides liquidating the investment and transferring the good across islands by themselves, have the additional possibility of having their claims to future consumption directly transferred to the bank in the other island. At time 2 the banks compensate their claims and transfer the corresponding amount of the good across space. The technology to transfer the good at time 2 is available for trades only between banks. Under certainty about investment returns, the claims just offset each other and in a netting system no liquidation takes place to satisfy the travelers’ consumption needs.

To summarize, early diers withdraw and consume at time 1. Compulsive travelers consume at time 2 but, under netting, also have the choice between withdrawing early and transferring the good themselves to the other island or transferring their bank accounts to the other island. Strategic travelers have the same options as the compulsive travelers along with the additional possibility to have their account untouched and to consume at time 2 at their own island.

Consumers have utility functions

\[
U(C_1, C_2, \hat{C}_2) = U(C_1) \quad \text{with probability } t
\]
\[
U(\hat{C}_2) \quad \text{with probability } (1 - t)(1 - \lambda)
\]
\[
U(C_2 + \hat{C}_2) \quad \text{with probability } (1 - t)\lambda \text{ with } C_2 \cdot \hat{C}_2 = 0,
\]
where $C_2$ denotes consumption at time 2 in the home island and $\hat{C}_2$ denotes consumption at time 2 at the other island. $U$ is a state-dependent utility function such that $U' > 0$, $U'' < 0$, $U''(0) = +\infty$ and with a relative risk aversion coefficient greater than or equal to one.\(^{10}\) The condition $C_2 \cdot \hat{C}_2 = 0$ forbids consumption in both islands.

The structure of the economy and the agents’ ex ante utility functions are common knowledge.

**Strategies**

Denote by $\Psi$ the set of late diers’ types; i.e., $\Psi = \{\text{ST, CT}\}$, where ST stands for strategic traveler, and CT for compulsive traveler. In any given candidate equilibrium, the deposit contract offers a consumption profile at time 1 and time 2 which is a function of the actions of the depositors in both islands (Run, Travel, or Wait). Early diers withdraw at time 1 and do not act strategically. Late diers behave strategically, playing a simultaneous game at time 1. We now introduce some notation we use for the rest of the paper. The strategic travelers’ set of actions is $S = \{W, T, R\}$, where W stands for waiting and withdrawing at time 2, T for traveling and having your claims transferred to the other island, and R for running, that is, withdrawing at time 1 and storing the good if necessary. The compulsive travelers’ set of actions is $S' = \{T, R\}$. Since $S' \subseteq S$, whenever the strategic travelers choose an action in $S'$, the compulsive travelers will do the same.

A strategy $s_c$ is an element of the set of functions from $\Psi$ into $S$ for the strategic travelers, and into $S'$ for the compulsive travelers.

It is worth pointing out a difference in the interpretation of the time horizon between our model and that of D-D. In our model, the three periods of the D-D timing all take place within 24 h, when all transactions are executed. The costs associated with the liquidation of the investment can be interpreted as the interest differential between reserves and interest-bearing money market instruments.

As a benchmark, we now compare net and gross payment systems.

**Proposition 1.** Under certainty about investment returns,

(i) gross and net settlement systems yield the same allocations as two D-D economies with different fractions of early diers, $t + (1 - t)(1 - \lambda)$ for the former, and $t$ for the latter;

(ii) net settlement dominates gross settlement.

**Proof.** Point (i) is obvious from the above discussion. As for (ii), the fact that in a D-D economy the expected welfare decreases with the proportion of early diers is proved in the Appendix, although it is quite intuitive.□

\(^{10}\) We will drop the superscript “\(\ast\)” whenever this does not create ambiguity.
Since in a gross system, more consumers withdraw at time 1, a higher proportion of the investment is liquidated than under netting. Since the investment returns more than 1, a netting system dominates a gross system. As liquidation is costless, a higher proportion of liquidation is equivalent to a higher proportion of reserves in the banks’ portfolio. Proposition 1 is tantamount to saying that with certain investment returns, in a gross system banks would have to hold more reserves.

The intuition for our result is similar to that of the related papers by Bhattacharya and Gale (1987) and by Bhattacharya and Fulghieri (1994). Both papers study modified versions of D–D economies. Bhattacharya and Gale consider several banks with i.i.d. liquidity shocks in the sense that their proportion of early die is random. Bhattacharya and Fulghieri consider banks with i.i.d. shocks in the timing of the realized returns of the short term technology. A common theme of both papers is that if the shocks are observable and contractible, in the aggregate, liquidity shocks and timing uncertainty are completely diversifiable. Thus an interbank market (which in our model can be interpreted as a netting system) can improve upon the allocation with no trade between banks (in our model a gross system). The additional location risk we introduce with respect to D–D is also fully diversifiable. If it was not, the optimal allocation would be contingent on the aggregate risks, as Hellwig (1994) has shown for interest rate risk.

4. STOCHASTIC RETURNS AND INFORMED DEPOSITORS

The preceding analysis offers a benchmark to compare gross and net settlement systems. We now extend the basic setup to introduce contagion risk.

The investment return $\tilde{R}$ at time 2 is random, $\tilde{R} \in \{R_L, R_H\}$, where L and H stand for low and high respectively, $p_L$ denotes the probability of the low return (which we assume is “sufficiently” low in a sense to be defined precisely later), and $R_L < 1 < R_H$, $E\tilde{R} = p_L R_L + p_H R_H > 1$. At time 1 the late die in each island privately observe a signal $y_K \in \{y_L, y_H\}$, $K = L, H$, uncorrelated across islands, which fully reveals $\tilde{R}$. Ex ante the two banks offer the same contract.

**Strategies**

With stochastic returns, late die choose their actions as functions of the signal in both islands. A strategy $s_\alpha(y_K)$ is an element of the set of functions from $\Psi \times Y$ into $S$ for the ST, and into $S'$ for the CT.

A strategy profile is a set of strategies for each type of late die and for each signal-island pair. For example, in the strategy profile $\{(W_{ST}(y_H),
(T_{CT}(y_H), T_{ST}(y_L), T_{CT}(y_L))$ (which we will denote simply $[(W, T), (T, T)]$ whenever this is unambiguous), if the high signal is observed, the strategic travelers wait and withdraw at time 2 and the compulsive travelers travel: $(W, T)$. If the low signal is observed, all late diers travel: $(T, T)$.

Notice that with interim information, withdrawal at time 1 by late diers might be socially optimal (as in the model of information-based runs of Jacklin and Bhattacharya (1988) or Chari and Jagannathan (1988)), while it was never so in the deterministic case. Hence, runs have a disciplinary role because they trigger the closure of inefficient banks.\footnote{One criticism made to models of this type is to question why managers acting on behalf of their depositors would keep the bank operating when it is worth more dead than alive. In our model the answer relies on the fact that bad banks have an incentive to stay in business to free ride on the assets of the good ones through the payment system.}

For a given settlement system we can summarize the timing as follows. At time 0 a deposit contract is offered by the banks to consumers in the same island and the banks invest. At time 1 time preference shocks occur and are privately revealed to depositors; the realization of the investment return on each island is revealed to its residents; the late diers in the two islands then play a simultaneous game acting on the basis of this information; in each island the bank liquidates a fraction of the investment to reimburse the depositors withdrawing at time 1. At time 2 the investment matures and the proceeds are distributed according to the contracts.

**Gross Settlement**

In a gross system the banks are isolated, and thus contagion is absent. Hence, except for the bank run equilibrium in a high-signal bank, all outcomes are efficient. That is, investment in the low-signal bank is liquidated and investment in the high-signal bank is allowed to mature, so that the efficient decision regarding bank closure is always taken.

**Net Settlement**

An analysis of the net settlement is more complex and requires several additional assumptions that make explicit how claims are settled in case of bankruptcy. A bankruptcy occurs when a bank is not able to fulfill its time 1 or time 2 obligations from the deposit contract. We will use two simplifying rules.

**Bankruptcy Rule 1.** This rule defines the assets to be divided among claim holders. Claim holders have a right to all the banks asset’s, including those brought by incoming travelers at time 2. Under this rule a bankrupt bank at time 1 stays in business simply to receive assets brought by the incoming travelers at time 2 and to pay the late diers.
Bankruptcy Rule 2. This rule, which defines how to divide the assets among the claim holders, establishes the equal treatment of claim holders (no seniority, or discrimination of any kind).

Bankruptcy at time 2 is solved by dividing the assets at time 2 using Rule 2. However, bankruptcy at time 1 is far more involved. We have to determine how to treat agents that pretend to be early diers. Under Bankruptcy Rule 1, late diers are allowed to postpone their consumption and benefit from the high-return assets the incoming travelers might bring with them. But a low return at time 2 for the other bank will diminish the payments the incoming travelers bring. The above rules capture the notion that banks participating in a netting scheme agree to honor the other members' liabilities arising from automatic intraday credit lines due to netting.\(^\text{12}\)

Equilibrium Analysis

A strategy profile is an equilibrium if there is no unilateral incentive to deviate. We characterize the equilibria in a net system in the following proposition.

**Proposition 2.** Under netting there are two equilibria.

*Equilibrium 1.* \([(W, T), (T, T)]\) occurs if and only if the equilibrium expected payoff for a strategic traveler in the low-signal bank exceeds that from running, i.e., if and only if
\[
p_H U(C_A) + p_L U(C_L) > U(C_1),
\]
where \(C_1\) denotes first period consumption,
\[
C_L = \frac{(1-t)R_L}{1-t}
\]
denotes second period consumption when both banks experience the low signal and
\[
C_A = \frac{(1-t)R_H(2-\lambda) + R_L}{(1-t)(3-\lambda)}
\]
denotes second period consumption in the high-signal island when the other island experiences the low signal.

*Equilibrium 2.* \([(R, R), (R, R)]\). Regardless of the signal observed, it is optimal for all late diers to run.

\(^{12}\) A similar loss-sharing rule is adopted in C.H.I.P.S. when a participant fails to settle its obligations.
Furthermore there are two efficient outcomes, \([(W, T), (R, R)]\) and \([(T, T), (R, R)]\), which cannot be supported as equilibria.

For sufficiently low values of \(p_L\) equilibrium 1 dominates equilibrium 2.

**Proof.** See Appendix.

Proposition 2 is the central proposition of this paper. Both equilibria exhibit contagion, in the sense that the signal received in one island affects the behavior and hence the consumption of depositors in the other. In the first equilibrium contagion arises when the two islands receive different signals.\(^{13}\) Payoffs to the depositors are an average of the two banks’ returns, so that the bank with a high return pays its depositors less than it would in isolation. This is analogous to the domino effect whereby the low return of a bankrupt bank lowers the return of an otherwise solvent bank (potential contagion equilibrium).

The inefficiency stems from a loss of the disciplinary role of bank runs under net settlement. Depositors in the low-signal bank expect to free ride on the other bank. Hence, when the low signal is observed, all late diers travel instead of running. When both banks receive a low signal, both keep operating. The reason banks are not liquidated when both islands observe a low signal, is that each late dier, ignoring the signal of the other island, prefers the expected utility from traveling to a high- (low-) signal bank with probability \(p_H\) (\(p_L\)) to the certain utility from withdrawing.

There are two ways in which contagion may trigger bank runs in equilibrium 2. First, for some parameter values equilibrium 1 fails to exist. The potential free riding of the late diers in the low-signal island destroys equilibrium 1 and makes it optimal to run in the high-signal island. In this case runs occur in both islands. The run that occurs in the high-signal island is induced by the mere fear of what can happen at the settlement stage. Second, even for the parameter values for which equilibrium 1 exists, given a speculative run in the high-signal island, it is optimal to run in the low-signal island as well. Notice that no similar argument applies under a gross system.

In what follows we will mainly focus on equilibrium 1, assuming that \(p_L\) is sufficiently low so that equilibrium 1 is preferred.

**The Effect of Different Bank Sizes**

Proposition 2 allows us to compare payment systems when banks have the same size. Still, it is interesting to have some insight on the type of equilibrium we obtain when banks have different sizes. This can be captured

\(^{13}\) Notice that this is the outcome of an insurance mechanism and not a swap of agents’ spending patterns.
assuming a different depositors’ measure in the islands, with the ratio between the two sizes going to infinity in the limit. The strategy space is now more complex because the strategies are conditioned not only on the signal, but also on the size. Still, the analysis of the limit case is straightforward and revealing. The outcome is determined by the signal the large bank receives. The analog of equilibrium 1 in the asymmetric bank case in the limit is given by the following strategies: for the large bank [(W, T), (R, R)] and for the small bank [(W, T), (T, T)]. The intuition for this is that in the limit case the consumption levels in case of different signals in the two islands are equal to the consumption level in the large bank, which is either \( C_H \) or 1 depending on the signal there. Hence, the signal observed by the late diers in the small island have a negligible effect on their consumption. Under the maintained assumptions what they obtain in equilibrium, \( p_H U(C_H) + p_L U(1) \), exceeds what they obtain by withdrawing, which is \( U(C_i) \). This justifies the strategy of the small bank compulsive travelers regardless of their signal and the strategy of the strategic travelers in the low signal bank. As for the strategic travelers receiving a high signal in a small bank, by waiting they obtain \( C_H \) with certainty against \( C_H \) with probability \( p_L \) and 1 with probability \( p_H \).

For the large bank, the effect of the small bank is negligible. Therefore, the outcome is similar to what would occur with only one bank in the system. When the low signal is received, running is optimal because there is no possibility to free ride on the other bank’s high return. When the high signal is received, it is optimal not to withdraw, because \( U(C_H) \) is obtained.14

5. THE TRADE-OFF BETWEEN GROSS AND NET PAYMENT SYSTEMS

The previous results demonstrate that the benefits from netting stem both from the possibility to invest more and from allowing the travelers to share the high expected return of time 2. Its cost stems from the continued

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14 Between the symmetric case of Proposition 2 and the zero/infinite size case we consider, the equilibrium may be far more involved because pure strategy equilibria may fail to exist. To understand this, consider a sufficiently large bank for which it is no longer optimal to play the strategy of equilibrium 1, [(W, T), (T, T)], because in case of low signal the return would be too low. The alternative equilibrium in the limit case, [(W, T), (R, R)], is not a Nash equilibrium either. Once all depositors of the large bank observing a low signal are running, a zero measure of depositors of the large bank observing the low signal will prefer to travel thus obtaining \( p_H U(C_H) + p_L U(1) \), which is superior to the bank run outcome, \( U(1) \). As a consequence, the equilibrium is characterized by mixed strategies on behalf of large banks’ depositors facing a low signal. A proportion of these depositors will travel thus lowering the return from traveling until we obtain \( C_A = 1 \), which is the indifferent point.
Contagion and Efficiency in Interbank Payments

operation of inefficient banks, those that receive a low signal. It is therefore possible to analyze the trade off between gross and net payment systems in terms of their allocative efficiency and study how this trade off is altered when the characteristics of the economy change.

Let the superscripts G and N denote gross and net, respectively. Since with gross settlement there is no contagion or wealth transfers between banks, expected utility is

\[
EU^G(\cdot) = p_H \left\{ t + (1 - t)(1 - \lambda)U(C_1^G) + (1 - t)\lambda U\left( 1 - C_1^G[t + (1 - t)(1 - \lambda)] R_H \right) \right\} + p_L U(1),
\]

where \( C_1^G \) is the optimal ex ante time 1 consumption under gross. With net settlement expected utility is

\[
EU^N(\cdot) = tU(C_1^N) + (1 - t)[p_H U(C_1^N) + p_L U[C_1^L] U(C_A) + U(C_B)] + p_L U(C_1^N)
\]

where \( C_1^N \) is the optimal ex ante time 1 consumption under netting. \( C_1^N \) and \( C_1^L \) are the values of consumption when both banks receive a high or low signal, respectively, and \( C_A \) and \( C_B \) are, respectively, the values of consumption at islands A and B when the bank at A experienced a high signal and that at B a low signal.

To compare gross and net settlement systems we construct the difference in their expected utility, \( \Delta = EU^G(\cdot) - EU^N(\cdot) \). We establish the following results.

**Proposition 3.** A gross settlement system is preferred (\( \Delta > 0 \)) when there is (i) a high expected cost of keeping an inefficient bank open (low \( R_L \)), (ii) a small fraction of compulsive travelers (low \( 1 - \lambda \)), (iii) a low probability that the state of nature is high (low \( p_H \)).

**Proof.** See Appendix.

We now illustrate these trade-offs for the particular case of logarithmic utility. In this case, it is easily proved that the optimal contract for an isolated bank is \( C_1^* = 1, C_2^* = R_K, K = L, H \) and therefore speculative bank runs never occur.\(^{15}\) With gross settlement, expected utility is

\(^{15}\) The fact that \( C_1^* = 1 \) (contrary to the D–D model, where there is no role for intermediation when \( C_1^* = 1 \)) does not constitute a limitation of our analysis since we focus on information-based bank runs rather than speculative bank runs as in D–D.
18 FREIXAS AND PARIGI

$EU^G(\cdot) = (1 - t)p_H\ln(R_H)$.

With net settlement expected utility is

$EU^N(\cdot) = (1 - t)(p_H\ln(R_H) + p_Hp_L[\ln(C_A) + \ln(C_B)] + p_H^2\ln(R_L))$

$$= (1 - t) \left\{ p_H^2\ln(R_H) + p_Hp_L\left[ \ln\left( \frac{(1 - tC_A^H)(R_H(2 - \lambda) + R_L)}{(1 - t)(3 - \lambda)} \right) \right. \right.$$

$$+ \left. \ln\left( \frac{(1 - tC_A^H)(2R_L + (1 - \lambda)R_H)}{(1 - t)(3 - \lambda)} \right) \right\} + p_H^2\ln(R_L) \right\}.$$

Assuming $\lambda < p_H$, it is straightforward to show that $\partial\Delta/\partial R_H < 0$, $\partial\Delta/\partial R_L < 0$, $\partial\Delta/\partial \lambda > 0$, $\partial\Delta/\partial p_H < 0$. Figure 1 presents the limiting frontier $\Delta = 0$ which separates the values of $p_L$, $\lambda$, $R_L$, and $R_H$ for which each system is preferred.

6. POLICY IMPLICATIONS

Although our model simplifies many aspects of the payment systems, it is useful to evaluate the payment system design policy. In many countries, gross real-time payment systems have proliferated, largely due to reduced operating costs for the data processing technology. While this aspect is important, disregarding the opportunity costs of holding liquid assets leaves out an essential dimension of payment systems and results in inefficient system design. Because of the trade-off between liquidity costs and contagion risk highlighted in the model, efficiency effects depend on a full constellation of parameters in addition to the technological dimension.

From that perspective it is helpful to list the main features that have changed in the banking industry in recent years. These trends are well documented facts in the main industrialized countries.

16 This corresponds to equilibrium (1) in Proposition 2. Since $C_A^H = 1$, equilibrium (2) will only occur if the necessary condition of equilibrium (1) is not fulfilled.
FIG. 1. Main tradeoffs between gross and net payment systems. $G$: gross settlement is preferred; $N$: net settlement is preferred.

1. Diminished costs of information processing and transferring;
2. Increased probability of failure;
3. Increased concentration in the banking industry;
4. Increased number of transactions due to mobility;
5. Improved liquidity management.

The implications of our model for these observations are straightforward. The first three observations favor a gross payment system, while the final
two favor a net system. In our model we have deliberately abstracted from the reduction in the cost of information processing and simply assumed that it was zero. Given this, the model predicts that a gross payment system is efficient when the probability of bank failure is high. Were this probability to decline sufficiently in the future, a net system might again become more attractive.

Regarding concentration, in our model a larger concentration in the banking industry implies that there are fewer compulsive travelers since the probability of traveling to another branch of the same bank is larger. With fewer compulsive travelers, a gross system is preferred. Hence, increased concentration favors a gross payment system.

On the other hand, the enormous growth in the volume of transactions that the payment system now handles, as well as improved liquidity management techniques that increase the opportunity cost of holding reserves, makes a net system more attractive.

The fact that we have observed such extensive development of gross systems suggests that Central Banks weigh the absence of contagion more heavily in their objective functions than the opportunity cost of holding reserves. Whether this weighting scheme is socially optimal deserves further consideration. If managers were given sufficient incentives to close the inefficient banks, then netting would always dominate gross payments. Moreover, were payments to be backed by a portfolio of high quality loans as collateral, efficiency would be improved by providing a substitute for the loss of bank-run discipline. Finally, a gross system might encourage good banks to create clubs which netted among themselves in order to economize on liquidity. Entry into those clubs would be difficult since it would confer a competitive advantage on members. This final result is a matter of deep concern about the market structure of the banking industry in the next century.

7. EXTENSIONS

Our basic framework can be extended in various directions. First, one could examine how bank capital can be used to mitigate contagion risk. Since equity capital reduces risk-taking, regulatory theory has suggested

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17 The central banker’s preference for gross settlement is clearly stated by Greenspan (1996, p. 691): “Obviously a fully real-time electronic transaction, clearing and settlement system, for example one with no float that approximates the currency model, would represent, other things equal, the ultimate in payment system efficiency.”

18 Private clearinghouses are emerging in the foreign exchange market. Examples include Echo in London, MultiNet in the U.S., and the global clearing bank called Group of Twenty, which comprises 17 of the world’s largest banks and plans to create a 24-hr organization for foreign exchange settlements within the next few years. (See The Economist (1996a, p. 83) and The Economist (1996b, p. 18)).
imposing minimum capital requirements. In our model this would lead to two potential benefits. On the one hand, since, once we introduce capital, the set of contracts equity holders can offer managers is enriched, they can provide managers with a sufficient stake in bank capital to compensate for their private benefits from keeping an inefficient bank open. On the other hand, capital requirements reduce the probability of bank failure and this might decrease the cost of contagion for depositors. The benefits of these two effects must be weighed against the cost of raising bank capital.

Second, interbank suspension of convertibility might mitigate contagion risk in a net settlement system. Suppose the suspension mechanism is defined in such a way that the banks refuse to serve travelers from the other bank in excess of the proportion of compulsive travelers, \((1 - t)(1 - \lambda)\). This notion of suspension can be interpreted as bilateral credit limits (as in C.H.I.P.S.). Suspension does not affect the strategies of the travelers from the high-signal bank, but it affects those of the travelers from the low-signal bank. In fact it would ration the travelers from the low-signal bank and might force some of them to withdraw at time 1. Since this forces liquidation of some low-signal bank assets, it improves efficiency. This could support the efficient outcome, \([(W, T), (R, R)]\), as an equilibrium. If in addition to interbank suspension, we introduce intrabank suspension conditional on the good state as in Gorton (1985), we also eliminate any speculative bank run in the high-signal island.

Third, changes in bilateral credit limits could replace the lost disciplinary effect of bank runs if they resulted from peer monitoring (see Rochet and Tirole (1996a) for a theoretical discussion). In our model this extension is straightforward if we view monitoring as allowing a bank to observe the signal of the other bank. When monitoring costs are not “too high,” a disciplinary effect is introduced. A bank observing a bad signal on its counterpart will reduce its bilateral credit assessment to zero and force its counterpart to liquidate its assets. Inefficient banks disappear. Thus, absent asymmetric information about investment returns, a netting system dominates gross, as in Proposition 1.19

Fourth, we can analyze the role of collateral in securing payments in a net system. Notice that if cash is the only collateral asset, netting provides no gains over a gross payment system. Moreover, if we allow a portfolio of loans to be used as collateral, if valued at their nominal value, the effect is the same in the two systems because ex post the collateral value is insufficient. On the other hand, if the bank portfolio of loans could be used as collateral once the number of its traveling depositors was known, then

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19 If bilateral credit limits could be changed costlessly, the functions of the settlement system could be taken over by the interbank lending market. The difference from the interbank market resides in the existence of implicit automatic credit lines in a settlement scheme.
the inefficiency of netting would be resolved. A bank in which every deposi-
tor travels will not find sufficient collateral and will therefore be forced to
default. Since this happens only in the case of low returns, inefficient
banks disappear.

Fifth, we can also analyze the coexistence of net and gross payment
systems, a standard feature in most industrialized countries. Our model
implies that by combining the two systems in the right proportions it is
possible to improve efficiency. The source of inefficiency under netting is
the lack of the disciplinary role of runs. Consequently, by combining the
two systems, it is possible to preserve information-based runs (although
with fewer agents or smaller payments) while economizing on liquid assets.
Our model does not, however, predict that large-value transactions will be
executed through the gross settlement, a feature commonly observed. Only
if we make the ad hoc assumption that informed depositors make large
transactions would we conclude that it is efficient to channel them through
the gross payment system and net the small amounts.

8. CONCLUSIONS

In this paper we have modeled the impact of the payment system on the
risks and the returns in the banking industry. Our analysis establishes the
trade-off between real-time gross settlement (RTGS) and netting in terms
of the necessary reserves and contagion risk. Second, it points out that the
disciplinary effect of bank runs may be lost in a netting system. Finally, it
allows to establish how regulation may improve upon both RTGS and
netting.

In this conclusion we discuss one final potential extension by focusing
on a particular application of our results to the analysis of the European
Monetary Union. The trade-off between risk and liquidity in gross and net
payment systems is one of the key factors in the design of the TARGET
(Trans-European Automated Real-Time Gross Settlement Express Trans-
fer) system for transactions in Euro.\footnote{The architecture of the payment
systems in the Euro area will be composed of the European System of Central
Banks (the European Central Bank and its regional offices corresponding
to the current national central banks) and of TARGET. TARGET establishes
the linkages between national RTGS systems. In TARGET each payment order is
immediately and irrevocably settled in central bank money by debiting and
crediting the banks’ accounts with the national central bank, as, e.g., in FEDWIRE.
The novelty of the mechanism, however, is that in TARGET when a bank from
country A sends a payment message to a bank in country B, bank A’s account with
its national central bank will be debited and bank B’s account with its national
central bank will be credited. Thus the two national central banks will
net their positions bilaterally each day. Intraday credit has a limited role in TARGET.
In fact, on the one hand, the national central banks will provide intraday credit to participants
in TARGET only by making use of two facilities, fully collateralized intraday overdrafts and}
RTGS international system, TARGET has been designed to minimize the systemic risk due to cross-border transactions. Our framework makes it possible to analyze this issue from the point of view of resource allocation and risk sharing. According to our model, where we reinterpret each bank as a National Central Bank, the choice of gross versus net depends on the comparison between the cost of holding reserves in accounts at the National Central Banks and the cost of losing the disciplinary effect of bank runs. It is reasonable to assume that the cost of holding reserves is low because the Euro is expected to be a hard currency with low interest rates. The cost due to the loss of disciplinary role is here more difficult to interpret. One possible interpretation is that one country in the Euro area is affected by a negative productivity shock and that, as a result, deposits fly to other countries. In this case the cost of the loss of disciplinary effect is high because, in addition to the cost of forbearance, there is a high political cost of transferring wealth from the supranational central bank (the European Central Bank) to a particular country. Furthermore, domestic banking authorities might have an additional incentive to keep a domestic bank open to free ride on sound foreign banks.

Finally, our model shows that an unintended consequence of weighing more safety than liquidity cost in TARGET might be that low-risk banks will benefit from developing their own private unsecured, reputation-based netting system to reduce the level of reserves they need. This may lead to an extension of the correspondence system if higher risk banks are short in the collateral required to obtain Central Bank overdrafts.

APPENDIX

Proof of Proposition 1. Assume first that the strategic travelers choose to consume at their home island. In a gross system the bank at a given island solves

\[
\max \ tU(C_1) + (1 - t)[\lambda U(C_2) + (1 - \lambda)U(\hat{C}_2)] \ w.r.t. \ \{C_1, C_2, \hat{C}_2, L\}
\]

s.t.

\[
itC_1 + (1 - t)(1 - \lambda)\hat{C}_2 = L \quad (A.1)
\]

Intraday repurchase agreements, which is essentially equivalent to the first. On the other hand, banks of European Union countries which are not part of the Euro area are excluded from the European Central Bank’s overdraft on the ground that an overdraft collateralized by assets denominated in a non-Euro currency entails an exchange rate risk. For an analysis of the main features of TARGET see European Monetary Institute (1996).
and

\[(1 - t)\lambda C_2 = R(1 - L), \quad (A.1')\]

which yields \(tU'(C_1) = \theta t\) and \((1 - t)(1 - \lambda)U'(\hat{C}_2) = \theta(1 - t)(1 - \lambda)\), where \(\theta\) is the Lagrange multiplier of the constraint (A.1), from which \(C_1 = \hat{C}_2\). Notice that \(C_2 > \hat{C}_2\). Therefore consuming at home is preferred by the strategic travelers.

In a net system consumption contracts in each island are \(C_1^*\) and \(C_2^*\), and banks store \(L^*\) and leave \(1 - L^*\) in the long-run technology, with \(tC_1^* = L^*\), \((1 - t)C_2^* = R(1 - L^*)\). From the first order condition \(U'(C_1^*) = RU'(C_2^*)\) it follows that

\[U'(L^*(t)/t) = R \cdot U'((1 - L^*(t))/R/(1 - t)). \quad (A.2)\]

Differentiating (A.2) w.r.t. \(t\), we have

\[U''(C_1) \left[\frac{-L^* + t dL^*/dt}{t^2}\right] = R^2 \cdot U''(C_2) \left[\frac{(-dL^*/dt)(1 - t) + (1 - L^*)}{(1 - t)^2}\right]\]

from which

\[\frac{dL^*/dt}{t^2} \left[\frac{U''(C_1)}{t} + \frac{R^2}{1 - t} U''(C_2)\right] = U''(C_1) \frac{L^*}{t} + R^2 \cdot \frac{(1 - L^*)}{(1 - t)^2} U''(C_2).\]

Since

\[\left[\frac{U''(C_1)}{t} + \frac{R^2}{1 - t} U''(C_2)\right] < 0 \quad \text{and} \quad U''(C_1) \frac{L^*}{t} + R^2 \cdot \frac{(1 - L^*)}{(1 - t)^2} U''(C_2) < 0,\]

then \(dL^*/dt > 0\). Since \(R(1 - L^*)\) is the return from the proportion of investment not liquidated, it follows that it declines with \(L^*\). Thus total welfare is reduced and consumption levels in both states are reduced.

Proof of Proposition 2. We sketch here the main argument. The rest of the proof follows exactly the same lines, so we have not detailed it. A complete proof is available from the authors upon request.

Strategy Profiles and Candidate Equilibria. Since the compulsive travelers’ strategy space is a subset of that of the strategic travelers, for each signal it must be the case that if the ST run, so must the CT and if the ST travel, so must the CT. As a result 16 strategy profiles are possible candidate
equilibria that we label conjectures. The home island is A unless otherwise specified. We use the following convention and notation.

<table>
<thead>
<tr>
<th>Number of conjecture</th>
<th>Signals</th>
<th>$y_H$</th>
<th>$y_L$</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>[W, T]</td>
<td>[R, R]</td>
<td>equilibrium 1</td>
</tr>
<tr>
<td>II</td>
<td>[T, T]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>III</td>
<td>[W, R]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>IV</td>
<td>[W, T]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>V</td>
<td>[W, R]</td>
<td>[R, R]</td>
<td></td>
</tr>
<tr>
<td>VI</td>
<td>[T, T]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>VII</td>
<td>[W, R]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>VIII</td>
<td>[W, T]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>IX</td>
<td>[R, R]</td>
<td>[R, R]</td>
<td>equilibrium 2</td>
</tr>
<tr>
<td>X</td>
<td>[T, T]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>XI</td>
<td>[W, R]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>XII</td>
<td>[W, T]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>XIII</td>
<td>[T, T]</td>
<td>[R, R]</td>
<td></td>
</tr>
<tr>
<td>XIV</td>
<td>[T, T]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>XV</td>
<td>[W, R]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>XVI</td>
<td>[W, T]</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

where, for example, conjecture I must be read as follows:

(W, T) means that when $y_H$ occurs it is optimal for strategic travelers to wait and for compulsive travelers to travel;

(R, R) means that when $y_L$ occurs it is optimal both for strategic travelers and for compulsive travelers to run;

and so on for the other conjectures.

Deposit Contracts. Consumption at the two times is a function of the strategies of the late diers in both islands, which are in turn a function of the signals in both islands. Let $C_i^1$ denote time 1 consumption under the candidate equilibrium corresponding to the strategy profile of conjecture $i = I, \ldots, XVI$. Define second period consumption when both banks receive the same signal $K$, under conjecture $i$, as

$$C_k^i = \frac{(1 - tC_i^1)R_k}{1 - t}, \quad K = L, H.$$

We will drop the superscript $i$ whenever this does not create ambiguity. As in D–D, we assume parameter values such that $C_i^1 > 1$.

Equilibrium Analysis. We now check the incentive to deviate unilaterally from the two equilibria which do exist (which correspond to conjectures
II and IX) and the two efficient outcomes that cannot be supported as equilibria (conjectures I and XIII) for a zero measure of late diers.

Conjecture I ([(W, T), (R, R)]). We will show that \( R_{ST}(y_L) \) (that is, running for a strategic traveler that has received signal \( y_L \)) is not the optimal strategy. Assume \( y_L \) is observed at A. If a low signal is observed at island B and all the late diers run, the return will be 1. With probability \( p_H \), a high signal is observed at island B, the strategic travelers at B wait and the compulsive travelers at B travel to A bringing assets \((1 - \tau)(1 - \lambda)C_H\). The expected payoff from a run is \( p_L U(1) + p_H U(C_R) \), where

\[
C_R = \frac{1 + (1 - \tau)(1 - \lambda)C_H}{1 + (1 - \tau)(1 - \lambda)}.
\]

When a zero measure of strategic travelers deviate to travel to island B they obtain \( p_H U(C_{11}) + p_L U(1) \). With probability \( p_H \) they end up in a high-signal bank at B and with probability \( p_L \) they end up at a low-signal bank at B where all the late diers run returning 1. Since \( p_L U(1) + p_H U(C_R) < p_L U(1) + p_H U(C_{11}) \), conjecture I is not an equilibrium, because late diers are better off deviating.

Conjecture II ([(W, T), (T, T)] is an equilibrium). Period 2 consumption may take the following values:

\[
C^H_t = \frac{(1 - \tau C^H_t)R_K}{1 - \tau}
\]

are the values if the banks at both islands experience the same signal \( K = L, H \); \( C_A \) and \( C_B \) are the values at islands A and B, respectively, if the bank at A experienced a high signal and that at B a low signal. To compute \( C_A \) and \( C_B \) consider the time 2 balance sheets of bank A with a high signal and of bank B with a low signal:

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>((1 - \tau C^H_t)R_H)</td>
<td>((1 - \tau)\lambda C_A) (ST)</td>
</tr>
<tr>
<td>((1 - \tau)C_B)</td>
<td>((1 - \tau)(1 - \lambda)C_A) (CT)</td>
</tr>
<tr>
<td></td>
<td>((1 - \tau)C_A) (Late diers from B)</td>
</tr>
<tr>
<td>((1 - \tau C^H_t)R_H + (1 - \tau)C_B)</td>
<td>(2(1 - \tau)C_A)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>((1 - \tau C^H_t)R_L)</td>
<td>((1 - \tau)C_B)</td>
</tr>
<tr>
<td>((1 - \tau)(1 - \lambda)C_A)</td>
<td>((1 - \tau)(1 - \lambda)C_A) (CT)</td>
</tr>
<tr>
<td>((1 - \tau C^H_t)R_L + (1 - \tau)(1 - \lambda)C_A)</td>
<td>((1 - \tau)(2 - \lambda)C_B)</td>
</tr>
</tbody>
</table>
From the above balance sheets we obtain two equations,

\[
(1 - tC_{II}^H)R_H + (1 - t)C_B = 2(1 - t)C_A \quad \text{and} \quad
(1 - tC_{II}^L)R_L + (1 - t)(1 - \lambda)C_A = (1 - t)(2 - \lambda)C_B,
\]

whose solutions are

\[
C_B = \frac{(1 - tC_{II}^H)((1 - \lambda)R_H + 2R_L)}{(1 - t)(3 - \lambda)} < C_A = \frac{(1 - tC_{II}^H)((2 - \lambda)R_H + R_L)}{(1 - t)(3 - \lambda)}
\]

since \( R_L < R_H \), where \( C_J, J = A, B \), are the claims of each depositor on bank \( J \)'s assets.

Finally to compute the optimal deposit contract one has to choose \( \{C_{II}^H, C_A, C_B, C_{II}^L, C_{II}^H\} \) to

\[
\max tU(C_{II}^H) + (1 - t)[p_H U(C_{II}^H) + p_L U(C_{II}^L) + p_H p_L (U(C_A) + U(C_B))] \ \text{s.t.}
\]

\[
C_{II}^H = \frac{(1 - tC_{II}^H)R_H}{1 - t}; \quad C_B = \frac{(1 - tC_{II}^H)((1 - \lambda)R_H + 2R_L)}{(1 - t)(3 - \lambda)};
\]

\[
C_A = \frac{(1 - tC_{II}^H)((2 - \lambda)R_H + R_L)}{(1 - t)(3 - \lambda)}
\]

which yields

\[
U'(C_{II}^H) = p_H U'(C_{II}^H)R_H + p_L U'(C_{II}^L)R_L
\]

\[
+ p_H p_L \left[ U'(C_B) \frac{(1 - \lambda)R_H + 2R_L}{3 - \lambda} + U'(C_A) \frac{(2 - \lambda)R_H + R_L}{3 - \lambda} \right].
\]

We want to prove that equilibrium 1 exists if and only if the condition we state in proposition 2 holds, namely

\[
p_H U(C_A) + p_L U(C_{II}^H) > U(C_{II}^H). \tag{A.3}
\]

Notice first that if (A3) holds, we also have

\[
p_H U(C_{II}^H) + p_H U(C_A) > U(C_{II}^H) \tag{A.4}
\]

and

\[
p_H U(C_{II}^H) + p_L U(C_B) > U(C_{II}^L). \tag{A.5}
\]
To check the optimality of $W_{ST}(y_H)$ we first compute the payoffs from the different strategies:

—The expected payoff from waiting is $p_H U(C_{II}^{H}) + p_L U(C_{A})$. Either the bank at island B has received a high signal and its late diers do not withdraw or it has received a low signal so that the return is $C_A$.

—Similarly, the expected payoff for a strategic traveler from a deviation to travel is $p_H U(C_{II}^{H}) + p_L U(C_{II}^{L})$. Therefore, for the strategy $W_{ST}(y_H)$ to be optimal, the expected payoff from waiting must exceed that from traveling, which is satisfied since $C_A \geq C_B$.

—The expected payoff for a strategic traveler from a deviation to running is $U(C_{II}^{H})$. The necessary condition for $W_{ST}(y_H)$ to be optimal is (A.4).

To check the optimality of $T_{ST}(y_L)$ notice first that the expected payoff from traveling is $p_H U(C_{A}) + p_L U(C_{II}^{I})$, which exceeds that from waiting $p_H U(C_{II}^{I}) + p_L U(C_{II}^{L})$ since $C_A > C_B$. Since the expected payoff from running is $U(C_{II}^{L})$, the necessary and sufficient condition for $T_{ST}(y_L)$ to be optimal is (A.3). Notice that if (A.3) is satisfied, both $T_{ST}(y_L)$ and $W_{ST}(y_H)$ are optimal.

To check the optimality of $T_{CT}(y_H)$ notice that it is the optimal strategy if (A.3) holds. Recalling that (A.3) implies (A.5) and observing that $C_B > C_{II}^{H}$, the expected payoff from traveling for a compulsive traveler, $p_H U(C_{II}^{H}) + p_L U(C_{B})$, exceeds that from a deviation to running, $U(C_{II}^{L})$.

To check the optimality of $T_{CT}(y_L)$, remark that the same conditions of $T_{ST}(y_L)$ apply because the compulsive travelers’ strategy set is a proper subset of the strategic travelers’ under $y_L$.

*Conjecture IX* ([{(R, R), (R, R)}] is an equilibrium). The proof is obvious. It corresponds to a speculative run in each island.

*Conjecture XIII* ([{(T, T), (R, R)}] is not an equilibrium). To show that $T_{ST}(y_H)$ is not the optimal strategy, notice that because of the bankruptcy procedure, in equilibrium the strategic travelers in the high-signal island receive less than $C_{II}^{H}$ in expected value. By waiting they receive $C_{II}^{H}$.

Using the same procedure one can show that the other conjectures cannot be supported as Nash equilibria, as there exist profitable deviations.

Finally we show that equilibrium 1 dominates equilibrium 2 for sufficiently low values of $p_L$. 
Since
\[ tU(C_H^1) + (1 - t)[p_H^tU(C_H^t) + p_L^tU(C_L^t) + p_H^t p_L^t(U(C_A) + U(C_B))] \]
\[ > tU(1) + (1 - t) \left[ p_H^tU(R_{H}) + p_L^tU(R_{L}) + p_H^t p_L^t \left( U \left( \frac{(1 - \lambda)R_H + 2R_L}{3 - \lambda} \right) \right) \right. \]
\[ + U \left( \frac{(2 - \lambda)R_H + R_L}{3 - \lambda} \right) \right] \]
we have that a sufficient condition for equilibrium 1 to dominate equilibrium 2 is that \( p_L^* < p_L^t \), where \( p_L^* \) solves
\[ \left[ (1 - p_L^*)^2U(R_{H}) + (p_L^*)^2(R_L) + (1 - p_L^*)p_L^* U \left( \frac{(1 - \lambda)R_H + 2R_L}{3 - \lambda} \right) \right. \]
\[ + U \left( \frac{(2 - \lambda)R_H + R_L}{3 - \lambda} \right) \right] = U(1). \]

**Proof of Proposition 3.** Using the envelope theorem we consider changes in \( \Delta \) close to the point \( \Delta = 0 \). (i) To show \( \partial \Delta / \partial R_L < 0 \) notice that \( EU^G(\cdot) \) does not depend on \( R_L \) and that \( EU^N(\cdot) \) is increasing in \( R_L \). (ii) To show \( \partial \Delta / \partial \lambda > 0 \) notice that from the FOC we have \( U'(C_H^1) = U'(C_H^0)R_H \), where \( C_H^0 \) is the optimal time 2 consumption in a gross system. Hence, from the assumption of relative risk aversion coefficient superior or equal to 1, it follows that \( C_H^0 \geq 1 \). Notice that
\[ \frac{\partial EU^G(\cdot)}{\partial \lambda} = p_H(1 - t) \left\{ U(C_H^0) - U(C_H^1) + U'(C_H^1)R_H \right\} \]
\[ \cdot \left[ -1 + tC_H^1 + (1 - t)\frac{C_H^0[\lambda + (1 - \lambda)]}{(1 - \lambda)} \right] \]
\[ = p_H(1 - t) \left\{ U(C_H^0) - U(C_H^1) + U'(C_H^1)R_H \frac{C_H^1 - 1}{(1 - t)\lambda} \right\} > 0. \]

On the other hand, since
\[ \frac{dC_A}{d\lambda} = \frac{(1 - tC_H^1)(R_L - R_H)}{[(1 - t)(3 - \lambda)]^2} < 0 \]
and
\[ \frac{dC_B}{d\lambda} = \frac{2(1 - tC_H^1)(R_L - R_H)}{[(1 - t)(3 - \lambda)]^2} < 0, \]
\[
\frac{\partial EU^N(\cdot)}{\partial \lambda} = p_H p_L (1 - t) \left[ U'(C_A) \frac{dC_A}{d\lambda} + U'(C_B) \frac{dC_B}{d\lambda} \right] < 0.
\]

Hence \(\partial \Delta / \partial \lambda > 0\). (iii) To show \(\partial \Delta / \partial p_H < 0\) is sufficient to observe that in the high-return state, expected utility is higher under the net system as shown in Proposition 1.

REFERENCES


