Curriculum Vitae

Giulio Peruginelli

April 5, 2019

Personal information

born in Livorno, Tuscany, Italy: 02 June 1979
Work-Address:

from March 2017: Via Trieste 63, Department of Mathematics of the University of Padova, Italy.

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Website: https://sites.google.com/site/giulioperuginelli/

Current Employment

From March 2017: Assistant professor (tenure track position) at the Department of Mathematics of the University of Padova, Italy.

Educational Qualifications

- 30 April 2004: graduated in Mathematics at the University of Pisa - record 110/110 cum laudae
  Thesis: "Teorema di irriducibilità di Hilbert e applicazioni al problema inverso di Galois" (Hilbert's Irreducibility Theorem and applications to the Inverse Galois Problem)
  Advisor: Prof. R. Dvornicich
- 1 January 2005- 13 December 2008: Phd Student, Department of Mathematics, University of Pisa.
  Thesis: Integer values of polynomials
  Advisor: Prof. U. Zannier
- 26 June 2003: Diploma in violin - Istituto Superiore di Studi musicali "P. Mascagni" - record 8.50/10.00

Research Interests

Image of polynomial mappings.
Parametrization of image set of polynomials.
Good reduction for elliptic curves and endomorphisms of the projective line.
Integer-valued polynomials over an algebra and study of the problems related to the additive and multiplicative structure of sich rings (regular basis, polynomial closure, prime spectrum, Prüfer properties).
Linear Algebra over Commutative Rings.

Abilitations

- Qualification in Maître de Conférences in the section 25 (Pure Mathematics) for the french universities, from 03/02/2016 to 31/12/2020.
- National Italian Abilitation for the position of associate professor in Algebra, starting from 28/3/2017.

Previous academic positions

1. January to March 2006: granted a Scholarship at the University of Bordeaux, France.

2. 9 June - 15 June, 30 June - 5 July 2008:
Visiting scholar of the Mathematics Section of the Abdus Salam International Centre for Theoretical Physics, Trieste, Italy. Website: http://www.ictp.it/.

3. 16 January - 7 March 2009: Visiting Scholar/Researcher at the Department of Mathematics of Purdue University, Lafayette, IN, USA.

4. 1 January - 30 April 2010: Post-Doc at the Hausdorff Research Institute for Mathematics of Bonn, Germany, for the HIM Junior Trimester Program on Algebra and Number Theory.
Website: http://www.hausdorff-research-institute.uni-bonn.de/.

5. 1 September - 31 December 2010: Chercheur de recherche associé (PostDoc) at the Laboratoire Amiénois de Mathématique Fondamentale et Appliquée, Faculté des sciences, Amiens, France, financed by the CNRS (Centre national de la recherche scientifique, http://www.cnrs.fr/).
Website: http://www.lamfa.u-picardie.fr.

6. January 2011 - December 2013: Post Doctoral Fellow at the Department of Mathematics of the Technische Universität of Graz, Austria.
Granted by the FWF (Austrian Science Fund), Project P23245-N18.
Website: https://www.math.tugraz.at/.

7. 14 March - 14 April 2011: Fellowship from the Mittag-Leffler Institute, Stockholm, Sweden.
Website: http://www.mittag-leffler.se/.

8. 15-22 April 2013: Invited guest at the Department of Mathematics of the Università di Roma 3, Italy, host: Prof.ssa F. Tartarone.

9. 1 January 2014 - 31 December 2015: Post-Doc at the Department of Mathematics of the University of Milano-Bicocca. Refused.

10. January 2014 - May 2014: Lecturer at the Department of Mathematics, University of Tennessee in Knoxville, TN, USA.

11. June 2014 - May 2016: Post-Doc at the Department of Mathematics of the University of Padova, Bando Giovani Studiosi, Assegno di Ricerca Senior.

12. 11-15 April 2016: Invited guest at the Department of Mathematics of the Technische Universität Graz, Austria, host: Prof.ssa S. Frisch.

13. August 2016 - February 2017, Post-Doc "Ing. Giorgio Schirillo" of the Istituto Nazionale di Alta Matematica (INDAM) at the Department of Mathematics of the University of Pisa.

14. 15-21 July 2018: Visiting Scholar at the Department of Mathematics, The Ohio State University, Columbus, invited by Prof. A. Loper.

15. 29 October-2 November 2018: Visiting Scholar at the Institut für Analysis und Zahlentheorie of the Technische Universität Graz, Austria, invited by Prof. S. Frisch.

16. 17-26 January 2019: Visiting Scholar at the Department of Mathematical Sciences, New Mexico State University, Las Cruces (USA), invited by Prof. B. Olberding.

Publications

- PhD Thesis in Mathematics, University of Pisa, Italy (2008).
  Title: "Integer values of polynomials".
  available online at: http://etd.adm.unipi.it/theses/available/etd-11282008-095329/

1. *Parametrizing over $\mathbb{Z}$ integral values of polynomials over* $\mathbb{Q}$, with U. Zannier.
Comm. Algebra, 38 (1) (2010), 119–130.
DOI: http://dx.doi.org/10.1080/00927870902855564.

2. *On some notions of good reduction for endomorphisms of the projective line*, with J.-K. Canci and D. Tossici, Manuscripta Math. 141 (2013), no. 1-2, 315-331.
DOI: http://dx.doi.org/10.1007/s00229-012-0573-y
Arxiv: http://arxiv.org/abs/1103.3853.

3. *Primary decomposition of the ideal of polynomials whose fixed divisor is divisible by a prime power*,
J. Algebra 398 (2014), 227-242,
DOI: http://dx.doi.org/10.1016/j.jalgebra.2013.09.016

4. *Integer-valued polynomials over matrices and divided differences*,
Monatsh. Math. 173 (2014), no. 4, 559-571.
DOI: http://dx.doi.org/10.1007/s00605-013-0519-9.
Arxiv: http://arxiv.org/abs/1301.6332.

5. *Integral-valued polynomials over sets of algebraic integers of bounded degree*,
J. Number Theory 137 (2014) 241-255.
DOI: http://dx.doi.org/10.1016/j.jnt.2013.11.007

6. *Factorization of integer-valued polynomials with square-free denominator*,
Comm. Algebra, 43 (1), 197-211, 2015.
DOI: http://dx.doi.org/10.1080/00927872.2014.897563.

7. *The ring of polynomials integral-valued over a finite set of integral elements*,
J. Commut. Algebra 8 (2016), no. 1, 113-141.
DOI: http://dx.doi.org/10.1216/JCA-2016-8-1-113
Arxiv: http://arxiv.org/abs/1411.1382

8. *Polynomial overrings of* Int($\mathbb{Z}$), with J.-L. Chabert,
J. Commut. Algebra 8 (2016), no. 1, 1-28.
DOI: http://dx.doi.org/10.1216/JCA-2016-8-1-1
Arxiv: http://arxiv.org/abs/1503.06035

9. *Properly Integral Polynomials over the Ring of Integer-valued Polynomials on a Matrix Ring*,
with N. J. Werner, J. Algebra 460 (2016), 320-339.
DOI: http://dx.doi.org/10.1016/j.jalgebra.2016.04.016

10. *The lattice of primary ideals of orders in quadratic number fields*, with P. Zanardo,
Int. J. Number Theory 12 (2016), no. 7, 2025-2040.
DOI: http://dx.doi.org/10.1142/S1793042117500737

11. *Non-triviality conditions for Integer-valued Polynomial Rings on Algebras*, with N. J. Werner,
Monatsh. Math. 183 (2017), no. 1, 177-189.
DOI: http://dx.doi.org/10.1007/s00605-016-0951-8
Arxiv: https://arxiv.org/abs/1604.06912

12. *Galois structure on integral-valued polynomials*, with Bahar Heidaryan, Matteo Longo,
J. Number Theory 171 (2017), 198-212.
DOI: http://dx.doi.org/10.1016/j.jnt.2016.07.007 (Open Access)

13. *Transcendental extensions of a valuation domain of rank one*,
Proc. Amer. Math. Soc. 145 (2017), no. 10, 4211-4226.
DOI: https://doi.org/10.1090/proc/13574
Arxiv: https://arxiv.org/abs/1611.00177

14. *Adelic versions of the Weierstrass approximation Theorem*, with J.-L. Chabert,
J. Pure Appl. Algebra 222 (2018), no. 3, 568-584.
DOI: http://dx.doi.org/10.1016/j.jpaa.2017.04.020
Arxiv: http://arxiv.org/abs/1511.03465

15. *Decomposition of Integer-valued polynomial algebras*, with N. J. Werner,
J. Pure Appl. Algebra 222 (2018), no. 9, 2562-2579.
DOI: https://doi.org/10.1016/j.jpaa.2017.10.007
Arxiv: http://arxiv.org/abs/1604.08337

16. *Maximal Subrings and Covering Numbers of Finite Semisimple Rings* with N. J. Werner,
Comm. Algebra, 46 (2018), no. 11, 4724-4738.
DOI: https://doi.org/10.1080/00927872.2018.1455099

17. *Prüfer intersection of valuation domains of a field of rational functions*,
J. Algebra 509 (2018), 240-262.
DOI: https://doi.org/10.1016/j.jalgebra.2018.05.012
Arxiv: https://arxiv.org/abs/1711.05485

Conference proceedings (with referee)

i. *Parametrization of integral values of polynomials*, Actes des rencontres du CIRM, 2 no. 2: Third International Meeting on Integer-Valued Polynomials, p. 41-49, 2010.
DOI: http://dx.doi.org/10.5802/acirm.32.

ii. *Integral closure of rings of integer-valued polynomials on algebras*, with Nicholas J. Werner, in "Commutative Algebra: Recent Advances in Commutative Rings, Integer-Valued Polynomials", M. Fontana, S. Frisch and S. Glaz (editors), Springer 2014, pp 293-305.
ISBN 978-1-4939-0924-7
DOI: http://dx.doi.org/10.1007/978-1-4939-0925-4_17
http://www.springer.com/mathematics/algebra/book/978-1-4939-0924-7.

iii. *Idempotent pairs and PRINC domains*, with L. Salce and P. Zanardo,
in "Multiplicative Ideal Theory and Factorization Theory - Commutative and Non-Commutative Perspectives", S. Chapman, M. Fontana, A. Geroldinger, and B. Olberding, Editors, Springer Verlag Publisher (2016), pp. 309-322. ISBN (Hardcover): 978-3-319-38853-3.
DOI:http://dx.doi.org/10.1007/978-3-319-38855-7_13
http://www.springer.com/it/book/9783319388533
Arxiv: http://arxiv.org/abs/1412.8089

Preprints

18. *The Zariski-Riemann space of valuation domains associated to pseudo-convergent sequences*, with Dario Spirito, submitted.
ArXiv: https://arxiv.org/abs/1809.09539

19. *Spectrum and Skolem properties of generalized rings of integer-valued polynomials*, in preparation.

- *Formal parametrizations of algebraic curves*, preprint Department of Mathematics University of Pisa, March 2008.

Invited talks, seminars

- 11 March 2005: University of Pisa, Department of Mathematics, Pisa, Italy.
- (**Invited**) 24 February 2009: Department Mathematics Colloquium, Purdue University, Lafayette, IN, USA.
- 24 June 2009: Doctoral program on Diophantine Geometry, Rennes, France.
- 8 July 2009: 26émes Journées Arithmétiques, Sant-Étienne, France.
- (**Invited**) 8 February 2010: Hausdorff Research Institute for Mathematics, Bonn, Germany.
- 20 May 2010: Commutative Ring Theory Days 2010, Roma, Italy.
- 12 July 2010: Department of Mathematics, Graz University of Technology, Austria.
- 11 October, 22 November and 6 December 2010: Department of Mathematics, University of Amiens, France.
- 29 November 2010: Troisiéme Rencontre sur les Polynomes à valeurs entières, Cirm, Marseille, France.
- (**Invited**) 29 March 2011: Institute Mittag-Leffler, Djursholm (Stockholm), Sweden.
- (**Invited**) 19 April 2011: Department of Mathematics, Università di Roma 3, Roma, Italy.
- (**Invited**) 12 May 2011: Department of Mathematics Karl-Franz University, Graz, Austria.
- 24 May 2011: 2nd Pohang International Conference on Commutative Algebras and Rings. Gyeongju City, South Korea.
- 27 June 2011: 27th Journée Arithmetiques, Vilnius, Lithuanie.
- 21 July 2011: Second International Conference and Workshop on Valuation Theory, Segovia, Spain.
- 16 September 2011: XIX Congresso UMI, Bologna, Italy.
- (**Invited**) 15 December 2011: Department of Mathematics Karl-Franz University, Graz, Austria.
- 5 June 2012: Commutative Rings and their Modules, 2012, Bressanone, Italy.
- (**Invited**) 15 April 2013: Department of Mathematics, Università di Roma 3, Italy.
- (**Invited**) 6 June 2013: Department of Mathematics, Karl-Franz University, Graz, Austria.
- (**Invited**) 5 February 2014: Department of Mathematics, University of Tennessee, Knoxville, TN (USA).
- (**Invited**) 22 March 2014: AMS Southeastern Spring Sectional Meeting, Knoxville, TN (USA).
- (**Invited**) 26 May 2014: Department of Mathematics, Università di Roma 3, Italy.
- 16 June 2014: ASTA 2014 - Algebraic Structures and Their Applications, Spineto (Siena), Italy.
- 22 September 2014: Arithmetic and Ideal Theory of Rings and Semigroups, University of Graz, Austria.

- 16, 23 and 30 October 2014: Department of Mathematics, University of Padova, Italy.
- (**Invited**) 5 November 2014: Padova-Verona MALGA Seminars, Department of Mathematics, University of Padova, Italy.
- (**Invited**) 19-20 March 2016: AMS Special Session on Commutative Ring Theory, State University of New York at Stony Brook, NY, USA.
- (**Invited**) 15 April 2016: Department of Mathematics, Technische Universität Graz, Austria.
- 15 February 2017: Department of Mathematics, University of Pisa, Italy.
- 6 June 2017: The Third Pohang International Conference on Commutative Rings and Algebras, Gyeongju, South Korea.
- (**Invited**) 4 December 2017: Department of Mathematics Università di Roma 3, Italy.
- (**Invited**) 17 March 2018: AMS Meeting, Special Session on Multiplicative Ideal Theory and Factorization (in honor of Tom Lucas retirement), Ohio State University, Columbus, Ohio, USA.
- 8 May 2018: Workshop on Valuation Theory, Department of Mathematics and Physics, University of Szczecin (Stettin), Poland.
- 27 June 2018: ALaNT 5 - Joint Conferences on Algebra, Logic and Number Theory, Mathematical Research and Conference Center in Bedlewo, Poland.
- (**Invited**) 18 July 2018: Department of Mathematics, Ohio State University, Columbus, Ohio.
- (**Invited**) 18 September 2018: Italian-Polish conference, Number Theory parallel session, Wroclaw, Poland.
- (**Invited**) December 2018: International Conference on Mathematics, Ho Chi Minh City, Vietnam.
- (**Invited**) 21 January 2019: Algebra Seminar, Department of Mathematical Sciences, New Mexico State University, Las Cruces (USA).
- (**Invited**) 25 January 2019: Departmental Colloquia, Department of Mathematical Sciences, New Mexico State University, Las Cruces (USA).
- (**Invited**) March 2019: AMS Meeting, Special Session on Valuations on Algebraic Function Fields and Their Subrings, University of Hawaii at Manoa, Honolulu, HI.

Organizing Activities

- Co-organizer (with L. Caputo) of the Seminar of Ph.D. Students in Algebra, Number Theory and Arithmetic Geometry at the Department of Mathematics, Università di Pisa (2005).
- Co-organizer (with S. Frisch and R. Rissner) of the conference "Commutative rings, integer-valued polynomials and polynomial functions" in Graz, Austria, 16-22 December, 2012.
Website: http://integer-valued.org/conf2012/.

Professional Activities

- Referee for the following journals: American Mathematical Monthly, Bulletin of the London Mathematical Society, Communications in Algebra, Journal of Algebra, Journal of Algebra and its Applications, Monatshefte für Mathematik, Proceedings of the American Mathematical Society.
- Reviewer for Zentralblatt MATH and Math Review.
- Reviewer for the Natural Sciences and Engineering Research Council (NSERC) of Canada.

Research projects - Grants

- **Title of the project**: "Integer-valued Polynomials".
Funded by: FWF (Fonds zur Förderung der wissenschaftlichen Forschung), Austria.
Length: 3 years, January 2011-December 2013.
Principal Investigator and Supervisor: Prof. Sophie Frisch (Technische Universität Graz).
- **Title of the project**: "Rings of Integer-Valued Polynomials on Algebras".
Funded by: Università di Padova, Italy.
Length: 2 years, June 2014-May 2016.
Principal Investigator: Giulio Peruginelli.
Scientific Supervisor: Prof. Luigi Salce (Università degli Studi di Padova).
Additional grant (12.000 €) for the best scientific project.

Workshop, Conferences and Schools attended

- **2005**
  - 12 April - 22 July: Research Program on Diophantine Geometry, Centro De Giorgi - Pisa, Italy.
- **2008**
  - 1-6 June: Number fields, Lattices and Curves, Cetraro, Italy, organized by GTEM (Galois Theory and Explicit Methods).
Website: http://www.mat.uniroma2.it/∼eal/gtem.html.
  - 18-28 June: Foundations of Computational Mathematics, City University of Hong Kong, China, organized by the Society for Foundations of Computational Mathematics. Website: http://www.focm.net.
  - 1 September - 15 November: Groups in Algebraic Geometry, Centro de Giorgi, Pisa, Italy.
- **2009**
  - 19-24 April: Advances in number theory and geometry, Verbania, Italy, organized by RISM (Riemann International School of Mathematics).
Website: http://www.mate.polimi.it/rism/.
  - 14-26 June: Doctoral program on Diophantine Geometry, Rennes, France, organized by Centre Emile Borel.
Website: http://ceb-dio2009.math.univ-rennes1.fr/index-fr.html.
  - 6-10 July: 26émes Journées Arithmétiques, Sant-Étienne, France, organized by Université de Saint-Étienne.
Website: http://ja2009.univ-st-etienne.fr/home.html.

- **2010**
  - 8-14 March: Italy-India Conference on Diophantine and Analytic Number Theory, Centro De Giorgi, Pisa, Italy.
    Website: http://www.crm.sns.it/hpp/events/event.html?id=115
  - 19-21 May: Commutative Ring Theory Days 2010, Dipartimento di Matematica Università di Roma Tre, Roma, Italy.
  - 25-29 May: Workshop on Rational Points - Theory & Experiment, Institute for Mathematical Research (FIM), ETH Zürich, Switzerland.
    Website:http://www.rationalpoints.ch/.
  - 31 May - 4 June: School on Local Rings and Local Study of Algebraic Varieties, ICTP, Trieste, Italy.
    Website: http://math.ictp.it/.
  - 6-10 September: 13th Mons Theoretical Computer Science Days, University of Picardie Jules Verne, Amiens, Francia.
  - 29 November - 3 December: Troisiéme Rencontre sur les Polynomes à valeurs entières, Cirm, Marseille, France.
    Website: http://www.lamfa.u-picardie.fr/evrard/colloqueIVP/index.htm.
  - 13 December: "Arithmétique Lille-Littoral", Lille, Francia.
- **2011**
  - 2-4 March: Winter School, Heights and Algebraic Numbers, Eberhard-Karls-Universität Tübingen, Germany.
  - 14 March-14 April: Algebraic Geometry with a view towards applications, Mittag-Leffler Institute, Djursholm (Stockholm), Sweden.
    Website: http://www.mittag-leffler.se/programs/current/1011s/.
  - 23-28 May: 2nd Pohang International Conference on Commutative Algebras and Rings. Gyeongju City, South Korea.
    Website: http://math.postech.ac.kr/.
  - 27 June - 1 July: 27th Journées Arithmétiques, Vilnius, Lithuania.
    Website: http://www.ja2011.lt/.
  - 18 July - 22 July: Second International Conference and Workshop on Valuation Theory, Segovia, Spain.
    Website: http://www.singacom.uva.es/oldsite/seminarios/ConfWorkVT/index.php
  - 12-17 September: XIX Congresso Unione Matematica Italiana, Bologna, Italy.
    Website: http://umi2011.dm.unibo.it/
- **2012**
  - 4-8 June: Commutative Rings and their Modules, Bressanone, Italy.
    Website: http://conference-bressanone2012.blogspot.com/
  - 16-22 December: Commutative rings, integer-valued polynomials and polynomial functions, Graz, Austria (co-organizer).
    Website: http://integer-valued.org/conf2012/.
- **2014**
  - 21-23 March: Southeastern Spring Sectional AMS Meeting, University of Tennessee, Knoxville, TN.
    Website: http://www.ams.org/meetings/sectional/2216_program.html

- 16-20 June: ASTA 2014 - Algebraic Structures and Their Applications, Spineto (Siena). - With a day dedicated to Alberto Facchini on the occasion of his 60th birthday.
  Website: http://events.math.unipd.it/asta2014/
- 22-26 September: Arithmetic and Ideal Theory of Rings and Semigroups, University of Graz in Graz, Austria, with a one-day special session dedicated to Franz Halter-Koch on the occasion of his 70th birthday. Website: http://math.uni-graz.at/ideals2014/.

- **2016**
  - March 19-20: AMS Special Session on Commutative Ring Theory, State University of New York at Stony Brook, Stony Brook, NY.
    Website: http://www.ams.org/meetings/sectional/2234_program_ss7.html
- **2017**
  - October 12: Conference on the occasion of Marco Fontana's retirement, Università degli Studi Roma Tre, Roma.
    Website: https://conference-fontana-day.blogspot.it/
- **2018**
  - March 18-19: AMS Meeting, Special Session on Multiplicative Ideal Theory and Factorization (in honor of Tom Lucas retirement), Ohio State University, Columbus, Ohio, USA.
    Website: http://ams.org/meetings/sectional/2250_program_ss7.html
  - May 7-12: Workshop on Valuation Theory, Department of Mathematics and Physics, University of Szczecin, Poland.
    https://math.usask.ca/fvk/WS2018.htm
  - June 24-29: ALaNT 5 - Joint Conferences on Algebra, Logic and Number Theory, Mathematical Research and Conference Center in Bedlewo, Poland.
    http://alant.math.us.edu.pl/
  - September 17-20: Italian-Polish conference, Number Theory parallel session, Wroclaw, Poland.
    http://umi-simai.ptm.org.pl/
  - December 18-20: International Conference on Mathematics, Ho Chi Minh City, Vietnam.
    http://icm2018.tdtu.edu.vn/

Teaching Experience

- Naval Academy, Livorno, Italy
  Website: http://www.marina.difesa.it/accademia/index.asp
  September-October 2004, Teaching (short term contract)
  Main activities and responsibilities: Teaching and examining students
- Naval Academy, Livorno, Italy
  Website: http://www.marina.difesa.it/accademia/index.asp
  September-October 2005, Teaching (short term contract)
  Main activities and responsibilities: Teaching and examining students
- University of Pisa, Faculty of Engineering, Italy
  November 2007-February 2008
  Teaching Assistant - Course of Linear Algebra
  Main activities and responsibilities: Teaching, tutoring and examining students

- University of Pisa, Faculty of Engineering, Italy
  February 2008-June 2008
  Teaching Assistant - Course of Theory of distributions and Theory of systems
  Main activities and responsibilities: Teaching, tutoring and examining students
- University of Pisa, Faculty of Engineering, Italy
  November 2008-July 2009
  Teaching Assistant - Course of Linear Algebra and Analysis
  Main activities and responsibilities: Teaching, tutoring and examining students
- University of Pisa, Faculty of Engineering, Italy
  April 2009-July 2009
  Teaching Assistant - Course of Theory of distributions and Theory of systems
  Main activities and responsibilities: Teaching, tutoring and examining students
- University of La Spezia "G. Marconi", Italy
  September 2009
  Teaching (short term contract)
- Department of Mathematics, Technische Universität, Graz, Austria, Winter semester 2011-2012.
  Einführung in algebraischer Kurven (Introduction to Algebraic Curves, in English).
  Program course: https://sites.google.com/site/giulioperuginelli/home/teaching/programmaCurve.pdf.
- Department of Mathematics, Technische Universität, Graz, Austria, Winter semester 2012-2013.
  Ausgewählte Kapitel der Algebra und Grundthemen Algebra (Selected Topics of Algebra and Advanced Algebra for PhD and Master students, in English).
  Program course: https://sites.google.com/site/giulioperuginelli/home/teaching/programmaAlgebra.pdf.
- Department of Mathematics, University of Tennessee, Knoxville, TN, USA.
  Spring Semester 2013-2014. Instructor for the following undergraduate courses (46 hours each):
  - Math 231 Ordinary Differential Equations I.
  - Math 251 Matrix Algebra I.

  Program of the courses available at: https://sites.google.com/site/giulioperuginelli/home/teaching.
- Department of Statistics, Università di Padova.
  Spring Semester 2016-17 and 2017-18. Linear Algebra.
- Department of Engineering, Università di Padova.
  Spring Semester 2017-18. Linear Algebra.

Description of the most significant papers

1. *Parametrizing over $\mathbb{Z}$ integral values of polynomials over $\mathbb{Q}$*

   with U. Zannier. Comm. Algebra, 38 (1), 119–130, 2010. http://dx.doi.org/10.1080/00927870902855564

   Given a polynomial $f(X)$ with rational coefficients we investigate the conditions under which the values attained by $f(X)$ over the integers are parametrizable by a polynomial with integer coefficients in one or possibly several variables. That is, given $f \in \mathbb{Q}[X]$ we look for a polynomial $g \in \mathbb{Z}[X_1, \ldots, X_m]$, for some $m \in \mathbb{N}$, such that $f(\mathbb{Z}) = g(\mathbb{Z}^m)$. Obviously, such a polynomial $f(X)$ has to be integer-valued, that is $f(\mathbb{Z}) \subset \mathbb{Z}$. Using Hilbert Irreducibility Theorem, we give an exhaustive classification of such polynomials $f(X)$: they are of the form $F(sX(sX-r))/2$, for some $F \in \mathbb{Z}[X]$ and s, r coprime odd integers and s a prime power (possibly equal to 1). In particular, only powers of 2 can appear in the common denominator of the coefficients of $f(X)$ and $f(X)$ satisfies the equation $f(X) = f(-X + r/s)$. Moreover, if such condition holds, we can parametrize $f(\mathbb{Z})$ with a polynomial with integer coefficients in 1 variable if $s = 1$ or 2 variables otherwise. We also give a criterium which establishes whether a polynomial with rational coefficients is of the above form.

2. *On some notions of good reduction for endomorphisms of the projective line*

   with J.-K. Canci and D. Tossici, Manuscripta Math. 141 (2013), no. 1-2, 315-331, http://dx.doi.org/10.1007/s00229-012-0573-y.

   Let φ be an endomorphism of $\mathbb{P}^1_{\overline{\mathbb{Q}}}$ defined over a number field K. Given a discrete valuation v of K, we consider here two notions of good reduction of φ at v, called Standard Good Reduction (S.G.R., for short) and Critically Good Reduction (C.G.R.). If we consider the reduced map φ_v, in general its degree is smaller or equal to the degree of φ. We say that the map φ has S.G.R. at v if the degree of the reduced map φ_v is equal to the degree of φ. This notion is frequently used in the study of arithmetical dynamical systems, allowing to reduce a global problem to a local problem. The second notion of good reduction has been recently introduced by Szpiro and Tucker to prove a finitess result about equivalence classes of endomorphisms of the projective line. We say that φ has C.G.R. at v if every pair of ramification points of φ do not coincide modulo v and the same holds for every pair of branch points. As an application of their result, Szpiro and Tucker showed that their theorem implies the well-known Shafarevich-Faltings theorem about the finiteness of the isomorphism classes of elliptic curves defined over a number field K having good reduction outside a prescribed finite set of discrete valuations of K. Szpiro and Tucker gave some examples that these two notions are not equivalent. We prove here that if φ has C.G.R. at v and the reduced map φ_v is separable, then φ has S.G.R. at v.

3. *Primary decomposition of the ideal of polynomials whose fixed divisor is divisible by a prime power*

   J. Algebra 398 (2014), 227-242, http://dx.doi.org/10.1016/j.jalgebra.2013.09.016.

   Given a polynomial $f \in \mathbb{Z}[X]$, we study here the image set of $f(X)$ over the integers by considering the ideal of $\mathbb{Z}$ generated by the values of $f(X)$. This ideal is usually called the fixed divisor of $f(X)$. Given a positive integer m, we consider the ideal I_m of $\mathbb{Z}[X]$ made up by those polynomial whose fixed divisor is divisible by m. We easily show that we can reduce the study of this ideal to the case of m equal to a prime power p^n. We describe here explicitly the ideal I_{p^n} by means of its primary decomposition, given a set of generators of each of its primary components, which are permuted by $\mathbb{Z}[X]$-automorphisms. The set of these primary components turns out to be equal to the set of polynomial ideals which are congruent to zero modulo p^n on the residue classes modulo p. Further application of this result are in the study of the factorization in the ring $\mathrm{Int}(\mathbb{Z})$.

4. *Integer-valued polynomials over matrices and divided differences*

   Monatsh. Math. 173 (2014), no. 4, 559-571, http://dx.doi.org/10.1007/s00605-013-0519-9.

   Let D be an integrally closed domain with quotient field K. We denote by $M_n(D)$ the D-algebra of $n \times n$ matrices with entries in D. Given a polynomial $f(X) = a_0 + a_1X + \ldots + a_nX^n$ in $K[X]$, we say that $f(X)$ is integer-valued over $M_n(D)$ if, for all matrices $M \in M_n(D)$, the matrix $f(M) = a_0I + a_1M + \ldots + a_nM^n$, obtained by evaluating the polynomial $f(X)$ in M has entries in D. The set of all such polynomials is a subring of $K[X]$ containing $D[X]$, denoted by $\mathrm{Int}_K(M_n(D))$. We give here a new characterization of the polynomials of this ring in terms of their divided differences. We prove that a polynomial $f \in K[X]$ is in $\mathrm{Int}_K(M_n(D))$ if and only if, for each k less than n, the k-th divided difference of $f(X)$ is integer-valued on any subset of the roots of any monic polynomial over

D of degree n. More precisely, if $\{\alpha_0, \ldots, \alpha_k\}$ is such a subset and $\Phi^k(f)(X_0, \ldots, X_k)$ denotes the k-th divided difference of $f(X)$, we have $\Phi^k(f)(\alpha_0, \ldots, \alpha_k)$ is integral over the ring D (in particular, it belongs to the integral clousure of D in the field obtained from K by adjoining the integral elements $\alpha_0, \ldots, \alpha_k$). If we also assume that the intersection of the maximal ideals of finite index is (0), then we may check the previous conditions only at subsets of the roots of monic irreducible polynomial of degree n (that is, the elements $\alpha_0, \ldots, \alpha_k$ are conjugate over D). We note that this characterization for the integer-valued polynomials over matrices shows that the ring $\mathrm{Int}_K(M_n(D))$ is contained in the ring $\mathrm{Int}^{\{n-1\}}(D)$ of polynomials which are integer-valued with their divided differences up to the order n introduced by Bhargava some years ago to study the Banach spaces of n-times continuously differentiable functions on a compact subset of a local field.

5. *Integral-valued polynomials over the set of algebraic integers of bounded degree*

   J. Number Theory 137 (2014) 241-255. http://dx.doi.org/10.1016/j.jnt.2013.11.007

   In a recent article, Loper and Werner introduced the ring of integral-valued polynomials over the set of algebraic integers of degree bounded by a fixed integer n, namely the ring $\mathrm{Int}_{\mathbb{Q}}(\mathcal{A}_n) \doteqdot \{f \in \mathbb{Q}[X] \mid f(\mathcal{A}_n) \subset \mathcal{A}_n\}$, where $\mathcal{A}_n$ is the set of algebraic integers of degree bounded by a fixed integer n. Here we prove that the subset A_n of $\mathcal{A}_n$ made up by algebraic integers of degree exactly equal to n is polynomially dense, that is $\mathrm{Int}_{\mathbb{Q}}(A_n, \mathcal{A}_n) = \{f \in \mathbb{Q}[X] \mid f(A_n) \subset \mathcal{A}_n\} = \mathrm{Int}_{\mathbb{Q}}(\mathcal{A}_n)$. This means that if a polynomial $f \in \mathbb{Q}[X]$ maps every algebraic integer of degree n to an algebraic integer, then the same property for $f(X)$ holds when we consider algebraic integers of smaller degree. Following a criterion of Gilmer about polynomially dense subsets of the ring of integers of a number field, we also prove that a similar result holds for a fixed number field K of degree n over $\mathbb{Q}$: the subset $O_{K,n}$ of the algebraic integers of K of degree equal to n is polynomially dense in the ring of integers O_K of K, that is $\mathrm{Int}(O_{K,n}, O_K) = \mathrm{Int}(O_K)$.

6. *Factorization of integer-valued polynomials with square-free denominator*

   Comm. Algebra, 43 (2015), no. 1, 197-211, http://dx.doi.org/10.1080/00927872.2014.897563.

   It is well known that the ring of integer-valued polynomials over $\mathbb{Z}$, $\mathrm{Int}(\mathbb{Z}) \doteqdot \{f \in \mathbb{Q}[X] \,|\, f(\mathbb{Z}) \subset \mathbb{Z}\}$, is far from being a unique factorization domain. Given $f \in \mathbb{Q}[X]$, integer-valued over $\mathbb{Z}$, an irreducible factor of $f(X)$ as an element of the ring $\mathrm{Int}(\mathbb{Z})$ is obtained by considering in a suitable way the irreducible factors of the numerator of $f(X)$ and the irreducible factors of the common denominator of its coefficients (more precisely, we can write $f(X)$ in the form $\frac{g(X)}{d}$, for some uniquely determined $g \in \mathbb{Z}[X]$, $d \in \mathbb{Z} \setminus \{0\}$ in reduced form). We introduce here a new way to find the essentially different factorizations of a given $f \in \mathrm{Int}(\mathbb{Z})$ in the case when the common denominator d is square-free.

7. *The ring of polynomials integral-valued over a finite set of integral elements*

   J. Commut. Algebra 8 (2016), no. 1, 113-141, http://dx.doi.org/10.1216/JCA-2016-8-1-113

   Let D be an integrally closed domain with quotient field K and A a torsion-free D-algebra which is finitely generated as a D-module. In previous works (see 4. and ii.) we show that the ring $\mathrm{Int}_K(A) = \{f \in K[X] | f(A) \subseteq A\}$ contains the intersection of the rings $D[X] + \mu_a(X)K[X]$, for $a \in A$, where $\mu_a \in D[X]$ is the minimal polynomial of a over D. In some cases this containment is an equality, for example when $A = M_n(D)$ or $A = T_n(D)$). In general, given a monic non-constant polynomial $p(X)$ of $D[X]$, we remark that the ring $D[X]+p(X)K[X]$ is a pullback. In this work we determine the integral closure of this ring. Similarly to the setting in ii.), the integral closure of $D[X] + p(X)K[X]$ is given by the ring $\mathrm{Int}_K(\Omega_p, \overline{D})$ of polynomials in $K[X]$ which are integer-valued over the set of roots Ω_p of $p(X)$ considered in an algebraic closure $\widehat{K}$ of K (here $\overline{D}$ denotes the integral closure of D in $\widehat{K}$).

We also show that for a finite subset Ω of $\overline{D}$, the ring $\mathrm{Int}_K(\Omega, \overline{D})$ is Prüfer if and only if D is Prüfer, generalizing a classical result of McQuillan (who consider the case of Ω a finite subset of D). We also give a criterium to establish when a pullback $D[X] + p(X)K[X]$, $p \in D[X]$ monic and non-constant, is integrally closed.

8. *Polynomial overrings of* $\mathrm{Int}(\mathbb{Z})$

   con J.-L. Chabert, J. Commut. Algebra 8 (2016), no. 1, 1-28. http://dx.doi.org/10.1216/JCA-2016-8-1-1

   It is now a classical result (due to Cahen, Chabert and Brizolis) that the ring of integer-valued polynomials $\mathrm{Int}(\mathbb{Z}) = \{f \in \mathbb{Q}[X] \mid f(\mathbb{Z}) \subseteq \mathbb{Z}\}$ is a non-noetherian Prüfer domain. By means of some results by Gilmer and Heinzer, which in particular classify the overrings of a Prüfer domain, we give a complete classification of the polynomial overrings of $\mathrm{Int}(\mathbb{Z})$, that is, rings R which lie between $\mathrm{Int}(\mathbb{Z})$ and $\mathbb{Q}[X]$. It turns out that such a ring can be represented as a ring of integer-valued polynomial over a compact subset of $\widehat{\mathbb{Z}}$, the profinite completion of $\mathbb{Z}$ (i.e., a fundamental system of neighborhood of 0 is given by the non-zero ideals of $\mathbb{Z}$). We also describe the valuation overrings of a given polynomial overring R of $\mathrm{Int}(\mathbb{Z})$ and classify those which are minimal and superfluos. Furthermore, we give a necessary and sufficient condition for such a ring R so that it admits an irredundant representation as intersection of valuation overrings.

9. *Properly Integral Polynomials over the Ring of Integer-valued Polynomials on a Matrix Ring*

   con N. J. Werner, J. Algebra 460 (2016) 320-339. http://dx.doi.org/10.1016/j.jalgebra.2016.04.016

   Let D be a Dedekind domain with finite residue fields. Recently, it has been proved that for each $n \geq 2$, the ring $\mathrm{Int}_K(M_n(D))$ of integer-valued polynomials over the $n \times n$ matrix ring over D, is not integrally closed; its integral closure has been determined in the publication ii). In this work we exhibit a family a polynomials which are integral over $\mathrm{Int}_K(M_n(D))$ but do not belong to the ring itself. We also show how this family is related to the notion of P-sequence for $\mathrm{Int}_K(M_n(D))$ and its integral closure, in the case when D is a DVR. Finally, we generalize to the case of the matrix ring $M_n(D)$, $n > 1$, a classical result due to Dickson which describes the ideal of polynomials with integer coefficients whose values over $\mathbb{Z}$ are divisible by a prescribed k-th power of a prime p, where $k \leq p$.

11. *The lattice of primary ideals of orders in quadratic number fields*

    con P. Zanardo, Int. J. Number Theory 12 (2016), no. 7, 2025-2040, http://dx.doi.org/10.1142/S1793042117500737.

    In this paper we consider quadratic orders in number fields, that is a domain O whose integral closure D is the ring of integers of a number field $K = \mathbb{Q}(\sqrt{d})$, where $d \in \mathbb{Z}$ is square-free. We also suppose that the conductor ideal $\mathfrak{f} = fD$ is a prime ideal of O, where $f \in \mathbb{Z}$ is prime. Since O is a Noetherian domain of dimension 1, each of its ideals can be written uniquely as a product of primary ideals. Note that every ideal coprime to the conductor, called regular, has unique factorization into product of prime ideals. In particular, each regular primary ideal Q is equal to a power of its radical, so that the primary ideals with the same radical forms a chain. When the radical of Q is equal to the conductor, we obtain a complete description of the lattice of $\mathfrak{f}$-primary ideals, according to whether f is split, inert or ramified in D. In each of the three cases, it turns out that the lattice of $\mathfrak{f}$-primary ideals has a structure by layers.

12. *Galois structure on integral-valued polynomials*

    con B. Heidaryan, M. Longo, J. Number Theory 171 (2017), 198-212. http://dx.doi.org/10.1016/j.jnt.2016.07.007

Let K be a number field with ring of integers O_K. In this work we study the generalized rings of integer-valued polynomials $\mathrm{Int}_{\mathbb{Q}}(O_K) = \{f \in \mathbb{Q}[X] \mid f(O_K) \subseteq O_K\}$ introduced in 2012 by Loper and Werner (see also the publication 5.). By means of the Density Theorem of Tchebotarev (i.e., every finite Galois extension $K/\mathbb{Q}$ is uniquely determined by the set of primes $p \in \mathbb{Z}$ which split completely in O_K), we prove that if K_1, K_2 are two distinct finite Galois extensions of $\mathbb{Q}$ then the corresponding rings $\mathrm{Int}_{\mathbb{Q}}(O_{K_i})$, $i = 1, 2$, are distinct. when the Galois extension $K/\mathbb{Q}$ has only tame ramification, we also determine a regular bases for such a ring, that is, a sequence $\{f_n\}_{n\in\mathbb{N}} \subset \mathrm{Int}_{\mathbb{Q}}(O_K)$ which generates the ring as a $\mathbb{Z}$-module and such that $\deg(f_n) = n$ for each $n \in \mathbb{N}$.

13. *Transcendental extensions of a valuation domain of rank one*

Proc. Amer. Math. Soc. 145 (2017), no. 10, 4211-4226. https://doi.org/10.1090/proc/13574.

Let V be a discrete valuation domain of rank 1 with quotient field K and let $\pi \in V$ be a generator of the maximal ideal P of V. We give a complete and explicit characterization of the set of valuation domains W of the field of rational functions $K(X)$ which extend V, with the properties that the residue field degree is finite and $\pi W = M^e$, for some $e \geq 1$, where M is the maximal ideal of W. Such a valuation domain W is equal to $W = W_\alpha = \{\varphi \in K(X) \mid \varphi(\alpha) \in \overline{\widehat{V}}\}$, for some α in $\overline{\widehat{K}} \cup \{\infty\}$, where $\overline{\widehat{K}}$ is the algebraic closure of the P-adic completion of K, where $\overline{\widehat{V}}$ is the absolute integral closure of $\widehat{V}$, the completion of V. Moreover, for $\alpha, \beta \in \overline{\widehat{K}}$ we have $W_\alpha = W_\beta$ if and only if α are β conjugated over $\widehat{K}$. This result establishes a bijection between the set of valuation domains $\{W_\alpha \mid \alpha \in \overline{\widehat{K}}\}$ and the set $\mathcal{P}^{\mathrm{irr}}$ of irreducible polynomials over $\widehat{K}$. Finally, we show that the set $\mathcal{P}^{\mathrm{irr}}$ endowed with an ultrametric distance introduced by Krasner is homeomorphic to the topological space $\{W_\alpha \mid \alpha \in \overline{\widehat{K}}\}$ endowed with the Zariski topology.

ii. *Integral closure of rings of integer-valued polynomials on algebras*

with N. J. Werner, in "Commutative Algebra: Recent Advances in Commutative Rings, Integer-Valued Polynomials, and Polynomial Functions", M. Fontana, S. Frisch and S. Glaz (editors), Springer 2016, pp. 293-305, ISBN 978-1-4939-0924-7

Let D be an integrally closed domain with quotient field K. Let A be a torsion-free D-algebra that is finitely generated as a D-module. For every a in A we consider its minimal polynomial $\mu_a(X) \in D[X]$, i.e. the monic polynomial of least degree such that $\mu_a(a) = 0$. The ring $\mathrm{Int}_K(A)$ consists of polynomials in $K[X]$ that send elements of A back to A under evaluation. If D has finite residue rings, we show that the integral closure of $\mathrm{Int}_K(A)$ is the ring of polynomials in $K[X]$ which map the roots in an algebraic closure of K of all the $\mu_a(X)$, $a \in A$, into elements that are integral over D. The result is obtained by identifying A with a sub-D-algebra of the matrix algebra $M_n(K)$ for some n and then considering polynomials which map a matrix to a matrix integral over D. We also obtain information about polynomially dense subsets of these rings of polynomials.